Submitted to *Quantitative Economics*

Supplement to "Rising skill premium and the dynamics of optimal capital and labor taxation"

YI-CHAN TSAI
Department of Economics, National Taiwan University

C. C. YANG
Institute of Economics, Academia Sinica, National Chengchi University and Fengchia University

HSIN-JUNG YU
Central Bank, Republic of China(Taiwan)

In this online appendix, we first describe our computation method. Then we develop and resolve simplified versions of our Ramsey and Mirrlees tax problems, respectively. Finally, we report the decomposition of the skill premium based on the capital-skill complementary effect and the relative quantity effect in the model.

APPENDIX A: COMPUTATION METHOD

We compute the transitional dynamics of optimal taxation by using the following procedure:

1. Given the sequence $\{q_t\}$ with $q_0 = 1$, we compute the steady state of the optimal allocation for the Ramsey problem and that of the constrained efficient allocation for the Mirrlees problem at each q_t. We then use these steady-state allocations as an initial guess for computing the transitional dynamics.

2. In the case of the Ramsey problem, we solve for the system of nonlinear equations (21)-(26) plus the implementability conditions (18), the resource constraints (13), the restriction (20), and the FOC with respect to φ. In the case of the Mirrlees problem, we solve for the system of nonlinear equations (41)-(45) plus the equality of the IC constraint (39) and the resource constraints (13). We substitute the resulting allocations into the IC constraints (39) and (40) to make sure that the incentive compatibility constraints for skilled and unskilled households are not violated.

Yi-Chan Tsai: yichantsai@ntu.edu.tw
C. C. Yang: ccyang@econ.sinica.edu.tw
Hsin-Jung Yu: hjyu@mail.cbc.gov.tw

APPENDIX B: SIMPLIFIED MODELS

B.1 *Ramsey problem*

The Lagrangian for the Ramsey problem is as follows:

$$\max_{\{c_{it},N_{st},N_{ut},K_{et+1}\}} \sum_{t=0}^{\infty}\beta^t \sum_{i\in\{s,u\}}\psi^i\left[c_{it}-\chi\frac{N_{it}^2}{2}\right]$$

subject to the implementability conditions

$$+\Psi^s\sum_{t=0}^{\infty}\beta^t[c_{st}-\chi N_{st}^2-(A_{s0}+T)],$$

$$+\Psi^u\sum_{t=0}^{\infty}\beta^t[c_{ut}-\chi N_{ut}^2-(A_{u0}+T)],$$

the resource constraints

$$+\sum_{t=0}^{\infty}\beta^t\Gamma_t\left[\mu N_{ut}+(1-\mu)K_{et}^{\lambda}N_{st}^{1-\lambda}+\frac{K_{et}}{q_t}-\frac{K_{et+1}}{q_t}-C_t-G_t\right],$$

and the restriction that all households face the same labor marginal tax rate at any point in time

$$+\sum_{t=0}^{\infty}\beta^t\Upsilon_t\left[N_{st}-N_{ut}\xi_t\right],$$

where we have used the FOCs of households to derive the implementability conditions and the last constraint invokes (31).[1]

The resulting FOCs are:

$$\psi^i+\Psi^i=\Gamma_t, i=\{s,u\}, \tag{B.1}$$

$$\Psi^s+\Psi^u=0, \tag{B.2}$$

$$\chi\left(\psi^u+2\Psi^u\right)N_{ut}=\Gamma_t\mu-\Upsilon_t\xi_t, \tag{B.3}$$

$$\chi(\psi^s+2\Psi^s)N_{st}=\Gamma_t\mu\xi_t+\Upsilon_t\left[1-N_{ut}\frac{\partial\xi_t}{\partial N_{st}}\right], \tag{B.4}$$

$$\frac{\Gamma_t}{q_t}=\beta\left[\Gamma_{t+1}\left(\frac{1}{q_{t+1}}+r_{et+1}\right)-\Upsilon_{t+1}N_{ut+1}\frac{\partial\xi_{t+1}}{\partial K_{et+1}}\right], \tag{B.5}$$

where we have used (29) in the derivation of (B.4).

[1]We do not use the "market" weights $\{\varphi^i\}_{i=\{s,u\}}$ here because the utility function takes the quasilinear form.

We obtain from (B.1)-(B.2):

$$\Psi^s = \frac{\psi^u - \psi^s}{2}; \Psi^u = \frac{\psi^s - \psi^u}{2}; \Gamma_t = \frac{\psi^s + \psi^u}{2} \equiv \Gamma. \tag{B.6}$$

We also obtain[2]

$$\Upsilon_t = \frac{\mu \xi_t}{1+\lambda}[\psi^u(1-\tau_{L_t}) - \Gamma], \tag{B.7}$$

$$\Upsilon_t = \frac{\mu}{\xi_t}[\Gamma - \psi^s(1-\tau_{Lt})]. \tag{B.8}$$

Putting (B.7) and (B.8) together yields equation (33).

Using (31), (30), (B.6), (B.7) and (33), we have from (B.5)

$$r_{et+1} = \frac{1}{\beta q_t} - \frac{1}{q_{t+1}} + \left(\frac{N_{st+1}}{K_{et+1}}\right) \frac{\lambda \mu \xi_{t+1}}{1+\lambda} \left[\frac{\psi^u - \psi^s}{\psi^s + \frac{\xi_{t+1}^2}{1+\lambda}\psi^u} \right].$$

Substituting the above r_{et+1} in (32) and using (30) gives equation (34).

B.2 *Mirrlees problem*

The Lagrangian for the Mirrlees problem is as follows:

$$\max_{\{c_{it}, N_{st}, N_{ut}, K_{et+1}\}} \sum_{t=0}^{\infty} \beta^t \sum_{i\in\{s,u\}} \psi^i \left[c_{it} - \chi \frac{N_{it}^2}{2}\right]$$

subject to the IC constraint

$$+\Lambda \sum_{t=0}^{\infty} \beta^t \left[c_{st} - \chi \frac{N_{st}^2}{2} - c_{ut} + \chi \frac{1}{2}(\frac{N_{ut}}{\xi_t})^2\right],$$

and the resource constraints

$$+\sum_{t=0}^{\infty} \beta^t \Gamma_t \left[\mu N_{ut} + (1-\mu) K_{et}^{\lambda} N_{st}^{1-\lambda} + \frac{K_{et}}{q_t} - \frac{K_{et+1}}{q_t} - C_t - G_t\right].$$

The resulting FOCs are:

$$\psi^s + \Lambda = \Gamma_t; \psi^u - \Lambda = \Gamma_t, \tag{B.9}$$

[2]Using (29), (30) and (B.6), we have from (B.4)

$$\psi^u \chi N_{st} = \Gamma w_{st} + \Upsilon_t [1 + \lambda \frac{N_{ut}}{N_{st}} \xi_t],$$

which by (31) and (29) gives (B.7). Using (B.6), we have from (B.3)

$$\psi^s \chi N_{ut} = \Gamma \mu - \Upsilon_t \xi_t,$$

which by (31) and (28) gives (B.8).

$$-\psi^s\chi N_{st}+\Gamma_t w_{st}-\Lambda\left[\chi N_{st}+\chi\frac{N_{ut}^2}{\xi_t^3}\frac{\partial\xi_t}{\partial N_{st}}\right]=0, \tag{B.10}$$

$$-\psi^u\chi N_{ut}+\Gamma_t w_{ut}+\Lambda\chi\frac{N_{ut}}{\xi_t^2}=0, \tag{B.11}$$

$$\frac{\Gamma_t}{q_t}=\beta\left[\Gamma_{t+1}\left(\frac{1}{q_{t+1}}+r_{et+1}\right)-\Lambda\chi\frac{N_{ut+1}^2}{\xi_{t+1}^3}\frac{\partial\xi_{t+1}}{\partial K_{et+1}}\right]. \tag{B.12}$$

From (B.9), we have

$$\Lambda=\frac{\psi^u-\psi^s}{2};\Gamma_t=\frac{\psi^u+\psi^s}{2}\equiv\Gamma. \tag{B.13}$$

Using (46) and (B.13), we have equation (47) from (B.11). Using (30), (46) and (B.13), we have from (B.10)

$$1-\tau_s(t)=\frac{\Gamma}{\psi^s+\Lambda}+\frac{\Lambda\lambda(1-\tau_u(t))w_{ut}(N_{ut}/N_{st})}{(\psi^s+\Lambda)\,\xi_t^2 w_{st}}=1+\frac{(\psi^u-\psi^s)\lambda(1-\tau_u(t))}{(\psi^u+\psi^s)\xi_t^3}\frac{N_{ut}}{N_{st}},$$

which implies $\tau_s(t)<0$. From (46), we have $\frac{N_{ut}}{N_{st}}=\frac{(1-\tau_u(t))}{(1-\tau_s(t))}\frac{1}{\xi_t}$. Substituting this result in the above equation leads to

$$[1-\tau_s(t)]^2-[1-\tau_s(t)]-\frac{(\psi^u-\psi^s)\lambda(1-\tau_u(t))^2}{(\psi^u+\psi^s)\xi_t^4}=0,$$

which in turn leads to equation (48).[3]

Using (30) and (B.13), we have from (B.12)

$$r_{et+1}=\frac{1}{\beta q_t}-\frac{1}{q_{t+1}}+\frac{\lambda\chi(\psi^u-\psi^s)}{(\psi^u+\psi^s)}\frac{1}{\xi_{t+1}^2}\frac{N_{ut+1}^2}{N_{st+1}}\frac{N_{st+1}}{K_{et+1}}.$$

Substituting the above r_{et+1} in (32) and using (28), (30), (46) and (47) gives equation (49).

Appendix C: Evolution of skill premium

In the face of increasing q_t (ESTP), we report how $\ln\xi$ in (11) evolves over time under optimal taxation.

C.1 *Ramsey taxation*

The right-hand panel of Figure C.1 reports the evolution of K_e/N_s and N_u/N_s over time in the face of an increasing q_t. According to (11), while K_e/N_s is the sole allocation that determines the capital-skill complementarity effect of the skill premium, N_u/N_s is the sole allocation that determines the relative quantity effect of the skill premium. The evolutions of K_e/N_s and of N_u/N_s shown in the right-hand panel of Figure C.1 explain why the skill premium shown in the left-hand panel of Figure C.1 is on the rise. Simply put, the capital-skill complementarity effect dominates the relative quantity effect.

[3]The other root of the quadratic equation is ruled out since it implies that $\tau_s(t)\geq 0$.

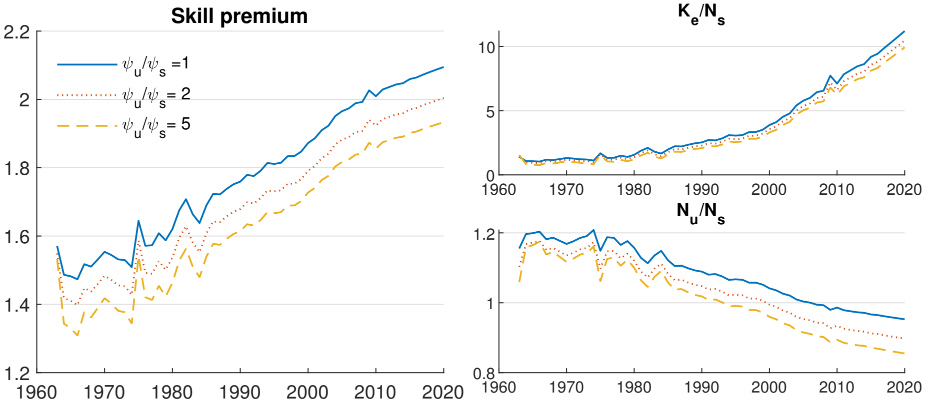

Figure C.1.: Log skill premium and allocations

C.2 *Mirrleesian taxation*

The figure for Mirrleesian taxation resembles Figure C.1 for Ramsey taxation.