Online Appendix to: "Consumption insurance with advance information"

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1 Risk sharing with private signals

Consider the two-period exchange economy described in the main body of the paper but with signals on agents' future income realizations that are only observed by the agents.[1] Let $c_{i,1}^{j}$ be first-period consumption of agents with reported private signal n^i and endowment e_j and $c_{i,2}^{jk}$ second-period consumption of agents with reported private signal n_i and endowment e_j in the first period and endowment e_k in the second period with $i, j, k \in \{l, h\}$. We focus on allocations with truthfully reported private signals. Let ν denote the precision of private signals.

The enforcement and resource feasibility constraints are given the same expressions as in the paper but with $\kappa = \nu$. Private information gives rise to another set of incentive constraints, truth-telling constraints that are given by the following expressions for high-income agents with a good and bad private signal

$$u(c_{h,1}^{h}) + \nu u(c_{h,2}^{hh}) + (1-\nu)u(c_{h,2}^{hl}) \geq u(c_{l,1}^{h}) + \nu u(c_{l,2}^{hh}) + (1-\nu)u(c_{l,2}^{hl}) \tag{1}$$
$$u(c_{l,1}^{h}) + \nu u(c_{l,2}^{hl}) + (1-\nu)u(c_{l,2}^{hh}) \geq u(c_{h,1}^{h}) + \nu u(c_{h,2}^{hl}) + (1-\nu)u(c_{h,2}^{hh}) \tag{2}$$

and for low-income agents with a good and bad private signal

$$u(c_{h,1}^{l}) + \nu u(c_{h,2}^{lh}) + (1-\nu)u(c_{h,2}^{ll}) \geq u(c_{l,1}^{l}) + \nu u(c_{l,2}^{lh}) + (1-\nu)u(c_{l,2}^{ll}) \tag{3}$$
$$u(c_{l,1}^{l}) + \nu u(c_{l,2}^{ll}) + (1-\nu)u(c_{l,2}^{lh}) \geq u(c_{h,1}^{l}) + \nu u(c_{h,2}^{ll}) + (1-\nu)u(c_{h,2}^{lh}) \tag{4}$$

An *efficient allocation* is a consumption allocation, $\{c_{i,1}^{j}, c_{i,2}^{jk}\}$, that maximizes ex-ante utility subject to the enforcement and resource constraints with $\kappa = \nu$ and truth-telling constraints (1)-(4).

With private information, consumption cannot be perfectly smoothed across states and both time periods conditional on the income-signal pair in the first period because of truth-telling. Agents with a low private signal are discouraged to report a high-signal type by threatening them with a particular low consumption for high-private signal households in case of a low income in the second period. To compensate for this lack of insurance, efficient allocations prescribe a high consumption in case of a high income in the second period to high-signal households. This however makes smoothing across states and time impossible.

As illustrated in Figure 1, we find that numerically increases in private-signal precision lead to qualitatively similar changes in unconditional moments and welfare as summarized in Proposition 1 for public signals. While welfare decreases, volatility of consumption increases when signals become more precise. Compared to public information, private information introduces additional welfare costs for informative signals. For this reason, welfare is lower and consumption is more dispersed with private than with public signals.

[1] Broer, Kapička, and Klein (2017) consider a limited commitment model in which household income is unobservable.

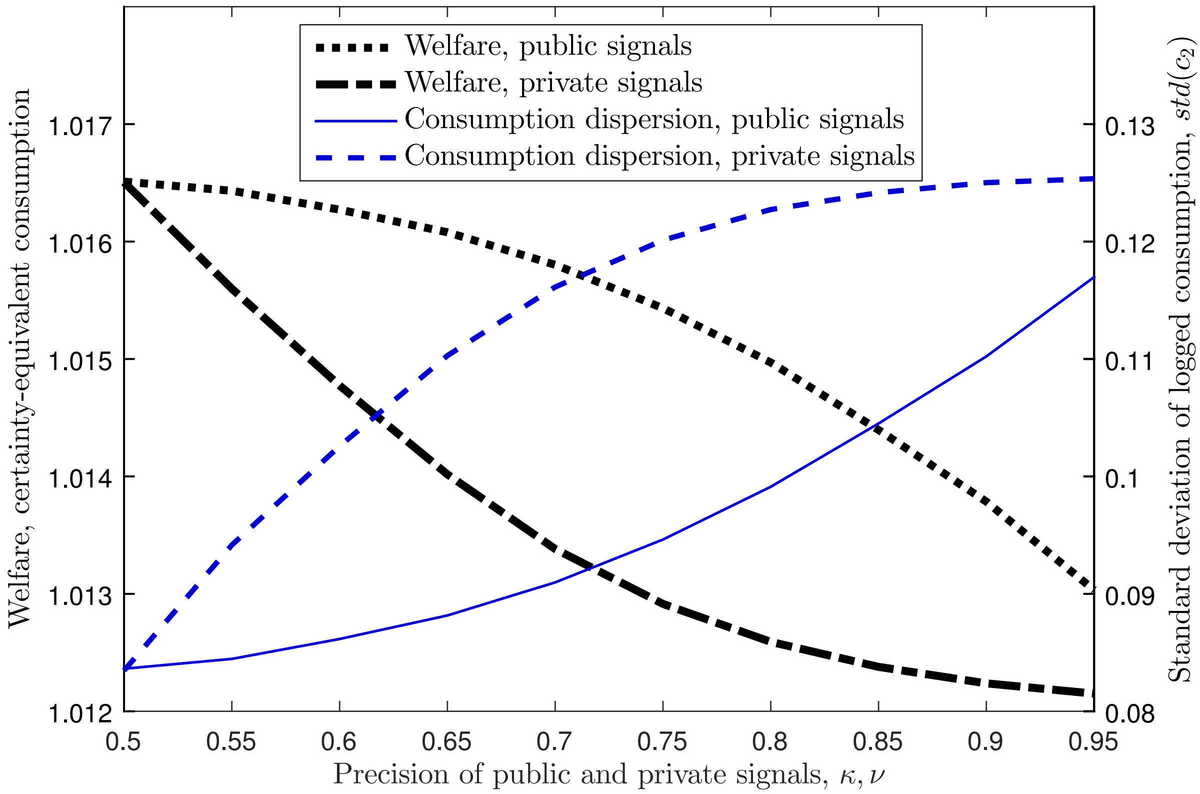

Figure 1: Two-period model. Welfare and consumption dispersion as functions of public and private information.

2 Persistent and transitory income shocks

In this section, we assume that logged income is the sum of a persistent and a transitory shock. We begin with stating equivalence results between the ARMA(1,1)-specification in the main text and the persistent–transitory income model. Afterwards, we develop an information environment in which households receive two signals; one signal informs about future realizations of the persistent while the other signal informs about the transitory shock. Moreover, we quantitatively study how risk sharing and insurance are affected by the two signals.

Equivalence between ARMA(1,1) and persistent–transitory shocks representation Log income of household i is modelled as the sum of persistent shocks z_{it} and transitory shocks ϵ_{it} as follows

$$\ln(y_{it}) = z_{it} + \epsilon_{it}, \qquad z_{it} = \rho z_{it-1} + \eta_{it},$$

where ϵ_{it} and η_{it} are independent, serially uncorrelated and normally distributed with variances σ_ϵ^2 and σ_η^2, respectively. As shown by Ejrnæs and Browning (2014), the ARMA(1,1) income process as specified in the main text permits a representation as an income process with orthogonal persistent and transitory shocks if and only if the autocorrelation coefficient ρ in both income processes is identical and $0 \leq \theta \leq \rho$. The variances of innovations in the persistent–transitory income specification are then given by

$$\sigma_\eta^2 = (\rho - \theta)\left(\frac{1}{\rho} - \theta\right)\sigma_u^2 \tag{5}$$

$$\sigma_\epsilon^2 = \frac{\theta}{\rho}\sigma_u^2. \tag{6}$$

We identify the parameters θ, σ_u^2 of the ARMA(1,1) from the cross-sectional within-group income variance and auto-covariance in the data, $\mathrm{var}[\ln(y_{it})]^d, \mathrm{cov}[\ln(y_{it}), \ln(y_{it-1})]^d$. This implies that for the parameter values as displayed in Table 1 in the main text, $\rho = 0.9989$ and $\sigma_\eta^2, \sigma_\epsilon^2$ chosen according to (5)-(6), the income specification with transitory and persistent shocks also matches the two data moments. More formally, the following holds

$$\mathrm{var}[z_{it}] = \frac{\mathrm{cov}[\ln(y_{it}), \ln(y_{it-1})]^d}{\rho}, \quad \sigma_\epsilon^2 = \mathrm{var}[\ln(y_{it})]^d - \mathrm{var}[z_{it}]. \tag{7}$$

Consider a discrete version of logged income in which the set of possible realizations for z_{it} is time-invariant and finite with N_z elements, $z_{it} \in Z \equiv \{z_1, ..., z_{N_z}\}$. The realizations are independent across households and evolve across time according to a first-order Markov process with time-invariant transition matrix $\pi_{jk}^z > 0$ for all j, k whose elements are the conditional probabilities of next period's shock $z' = z_k$ given current period shock $z = z_j$, $\pi(z' = z_k | z = z_j)$. The Markov chain induces a unique invariant distribution $\pi(z)$. The transitory shock is iid with two time-invariant realizations $\{\epsilon_1, \epsilon_2\} = \{-\sigma_\epsilon, \sigma_\epsilon\}$ that appear with equal probability and σ_ϵ as the standard deviation of the transitory shock. Households separately observe current and past realizations of both shocks. Log income is the sum of both shocks such that the set of realizations for income is also finite and time invariant, $y_{it} \in Y \equiv \{y_1, ..., y_N\}$, with $N = 2N_z$.

With innovations to persistent and transitory shocks and a single signal, it remains unclear on which part of income the signal is informative. For this reason, we set up an information environment with two signals, one that informs on future realizations of the persistent and one signal on future realizations of the transitory shock. As in the main text, we consider a hit-or-miss signal specification.

Signals on transitory and persistent shocks Let k_z, k_ϵ denote signals that inform on future realizations of the persistent shock z' and the transitory shock ϵ', respectively. Both signals have as many realizations as the shocks on which they inform. The precision of the signal on future transitory shocks is $\kappa_\epsilon = \pi(\epsilon' = \epsilon_j | k_\epsilon = \epsilon_j)$; uninformative signals exhibit precision 1/2, perfectly informative signals precision of one. The signal informs on future realizations of an iid shock that appear with equal probability which implies that the signal process that is consistent with household rationality is characterized by $\pi(k_\epsilon' | k_\epsilon) = \pi(k_\epsilon') = 1/2$ for all k_ϵ'. The precision of the signal on future persistent shocks is $\kappa_z = \pi(z' = z_i | k_z = z_i)$; uninformative signals exhibit precision $1/N_z$, while perfectly informative signals have a precision of one. The signal-conditional probabilities of future persistent shocks are given by the following

$$\pi\left(z'=z_k|z=z_i,k_z=k_j\right)=\frac{\pi^z_{ik}\kappa_z^{\mathbf{1}_{k=j}}\left(\frac{1-\kappa_z}{N_z-1}\right)^{1-\mathbf{1}_{k=j}}}{\sum_{z=1}^{N_z}\pi^z_{iz}\kappa_z^{\mathbf{1}_{z=j}}\left(\frac{1-\kappa_z}{N_z-1}\right)^{1-\mathbf{1}_{z=j}}} \quad . \tag{8}$$

The state vector is given by $s_{pt}=(z,\epsilon,k_z,k_\epsilon)$, where the subscript "pt" stands for "persistent–transitory". The conditional income probabilities read as follows

$$\begin{aligned}
\pi(y'|s_{pt}) &= \pi\left(z'=z_m,\epsilon'=\epsilon_n|z=z_i,\epsilon=\epsilon_j,k_z=z_k,k_\epsilon=\epsilon_l\right)\\
&= \pi\left(z'=z_m,\epsilon'=\epsilon_n|z=z_i,k_z=z_k,k_\epsilon=\epsilon_l\right)\\
&= \pi\left(z'=z_m|z=z_i,k_z=z_k\right)\times\pi\left(\epsilon'=\epsilon_n|k_\epsilon=\epsilon_l\right)\\
&= \frac{\pi^z_{im}\kappa_z^{\mathbf{1}_{m=k}}\left(\frac{1-\kappa_z}{N_z-1}\right)^{1-\mathbf{1}_{m=k}}}{\sum_{z=1}^{N_z}\pi^z_{iz}\kappa_z^{\mathbf{1}_{z=k}}\left(\frac{1-\kappa_z}{N_z-1}\right)^{1-\mathbf{1}_{z=k}}}\times\kappa_\epsilon^{\mathbf{1}_{n=l}}\left(1-\kappa_\epsilon\right)^{1-\mathbf{1}_{n=l}} \quad .
\end{aligned}$$

Even when the signals on future realizations of the persistent and the transitory are uninformative, agents possess advance information on future income given by the current realization of the persistent shock z. However, this 'signal' is only informative on the persistent shock such that even a unit-root process for z is only partially revealing future income.

The transition of the state s_{pt} with dimension $N_z\times 2\times N_z\times 2=4N_z^2=N^2$ follows from the conditional income probabilities

$$\begin{aligned}
\pi(s'_{pt}|s_{pt}) &= \pi\left(z'=z_m,\epsilon'=\epsilon_n,k'_z=z_o,k'_\epsilon=\epsilon_p|z=z_i,\epsilon=\epsilon_j,k_z=z_k,k_\epsilon=\epsilon_l\right)\\
&= \pi(k'_z=z_o|k_z=z_k)\pi(\epsilon_p)\pi\left(z'=z_k,\epsilon'=\epsilon_p|z=z_i,k_z=z_k,k_\epsilon=\epsilon_l\right)\\
&= \frac{\pi(k'_z=z_o|k_z=z_k)}{2}\frac{\pi^z_{im}\kappa_z^{\mathbf{1}_{m=k}}\left(\frac{1-\kappa_z}{N_z-1}\right)^{1-\mathbf{1}_{m=k}}}{\sum_{z=1}^{N_z}\pi^z_{iz}\kappa_z^{\mathbf{1}_{z=k}}\left(\frac{1-\kappa_z}{N_z-1}\right)^{1-\mathbf{1}_{z=k}}}\times\kappa_\epsilon^{\mathbf{1}_{n=l}}\left(1-\kappa_\epsilon\right)^{1-\mathbf{1}_{n=l}} \quad ,
\end{aligned}$$

with $\pi(\epsilon')=1/2$ for all ϵ', and a given Markov transition matrix for signals on persistent shocks P_{k_z} with the transition probabilities $\pi(k'_z=z_n|k_z=z_k)$ as elements chosen to satisfy the two requirements stated in the paper.

Mean-squared forecast errors In general, a given reduction in income risk resulting from conditioning income probabilities on signals can be due to signals on transitory or due to signals on persistent shocks. Unlike in case of a single signal, the reduction in the one-period ahead mean-squared forecast error for income as estimated by Dominitz (1998) in the data cannot uniquely identify signal precision in the model. To shed light on the question whether consumption insurance reacts more sensitively to advance information on future realizations of the persistent or the transitory shock, we proceed as follows.

As first exercise, we consider exclusively informative signals on the transitory shock, that is, $\kappa_z=1/N_z$ and $0.5\leq\kappa_\epsilon\leq 1$. The relevant reduction in the mean-squared forecast error for the transitory shock $\tilde{\kappa}_\epsilon(\kappa_\epsilon)$ is given by

$$\tilde{\kappa}_\epsilon(\kappa_\epsilon) = \frac{\text{MSFE}_\epsilon - \text{MSFE}_{\epsilon,k_\epsilon}(\kappa_\epsilon)}{\text{MSFE}_\epsilon}, \qquad 0 \leq \tilde{\kappa}_\epsilon(\kappa_\epsilon) \leq 1 \tag{9}$$

with

$$\begin{aligned}
\text{MSFE}_\epsilon &= \sum_\epsilon \pi(\epsilon) \sum_{\epsilon'} \pi(\epsilon'|\epsilon) \left[\epsilon' - \text{E}(\epsilon'|\epsilon)\right]^2 = \sigma_\epsilon^2 \\
\text{MSFE}_{\epsilon,k_\epsilon}(\kappa_\epsilon) &= \sum_{k_\epsilon} \pi(k_\epsilon) \sum_{\epsilon'} \pi(\epsilon'|k_\epsilon) \left[\epsilon' - \text{E}(\epsilon'|k_\epsilon)\right]^2 = 4\sigma_\epsilon^2(1-\kappa_\epsilon)\kappa_\epsilon \leq \sigma_\epsilon^2,
\end{aligned}$$

$\pi(k_\epsilon) = 1/2$ for all k_ϵ, $\pi(\epsilon') = \pi(\epsilon) = 1/2$ for all ϵ and

$$\pi\left(\epsilon' = \epsilon_n | k_\epsilon = \epsilon_l\right) = \kappa_\epsilon^{\mathbf{1}_{n=l}} \left(1-\kappa_\epsilon\right)^{1-\mathbf{1}_{n=l}}.$$

As second exercise, we assume that solely signals on the persistent shock are informative, that is, $\kappa_\epsilon = 0.5$ and $1/N_z \leq \kappa_z \leq 1$. It follows that the reduction in the mean-squared forecast error for future realizations of the persistent shocks $\tilde{\kappa}_z(\kappa_z)$ is given by the following

$$\tilde{\kappa}_z(\kappa_z) = \frac{\text{MSFE}_z - \text{MSFE}_{z,k_z}(\kappa_z)}{\text{MSFE}_z}, \qquad 0 \leq \tilde{\kappa}_z(\kappa_z) \leq 1 \tag{10}$$

with

$$\begin{aligned}
\text{MSFE}_z &= \sum_z \pi(z) \sum_{z'} \pi(z'|z) \left[y' - \text{E}(z'|z)\right]^2 \\
\text{MSFE}_{z,k_z}(\kappa_z) &= \sum_{z,k_z} \pi(z,k_z) \sum_{z'} \pi(z'|z,k_z) \left[z' - \text{E}(z'|z,k_z)\right]^2 \leq \text{MSFE}_z,
\end{aligned}$$

$\pi(z,k_z)$ as the joint invariant distribution of persistent shocks and signals on persistent shocks and the conditional probability $\pi(z'|z,k_z)$ as defined in (8).

Calibration and quantitative results For the quantitative exercise, we set the persistence parameter ρ equal to 0.9989. Given the persistence parameter, we identify the variances $\sigma_\epsilon^2, \sigma_\eta^2$ from the cross-sectional within-group income variance and auto-covariance in the CEX data as formalized in Equations (7). The method proposed by Tauchen and Hussey (1991) is used to approximate the persistent part of income by a Markov process with three states and time-invariant transition probabilities. As in the main text, we normalize the value of all income states such that mean income (or aggregate labor endowment) is equal to unity. In the following, we focus on the complete-markets model and study how advance information on transitory and persistent shocks affects consumption insurance.

We find that consumption insurance is not sensitive with respect to changes in the precision of signals on transitory shocks. Setting $\kappa_\epsilon = 0.95$ – corresponding to a reduction in the mean-squared forecast relative to uninformative signals $\tilde{\kappa}_\epsilon$ of over 80% – decreases the risk-sharing ratio merely by 0.04 and increases the regression coefficients by 0.01. As displayed in Table 1, a different picture emerges for the signals on persistent shocks. As can be seen in the second and

Table 1: Risk sharing, insurance and advance information: signals on persistent shocks

Risk-sharing ratio, RS			Regression coefficient, $\beta_{\Delta y}$		
$\tilde{\kappa}_z = 0.00$ (0.14)	$\tilde{\kappa}_z = 0.090$ (0.58)	**Data**	$\tilde{\kappa}_z = 0.00$ (0.14)	$\tilde{\kappa}_z = 0.082$ (0.57)	**Data**
0.94	0.60	**0.60**	0.01	0.11	**0.11**

Notes: ESC model. Informative signals on persistent shocks. Uninformative signals on transitory shocks. Risk-sharing ratio and regression coefficient in the data and in the model for different values of $\tilde{\kappa}_z(\kappa_z)$. Values for κ_z are in parentheses. Uninformative signals on persistent shocks, $\tilde{\kappa}_z = 0.00$, $\kappa_z = 0.14$, $N_z = 7$ and $N = 14$. Informative signals, $N_z = 3$ and $N = 6$.

fifth column, the risk-sharing ratio and the regression-coefficient from the data are captured with signals of precision $\tilde{\kappa}_z = 0.09$ and $\tilde{\kappa}_z = 0.08$, respectively.

The logic for the higher sensitivity of consumption insurance with respect to signals on persistent shocks is as follows. Signals on persistent shocks have a larger impact on consumption insurance than signals on transitory shocks because they are persistent. A high signal realization today not only means that a high realization of the persistent shock is more likely tomorrow but also that a high signal realization tomorrow is more likely. These long-lasting effects of a one-time signal realization result in tighter solvency limits and less consumption insurance. Signals on transitory shocks on the other hand only affect conditional shock probabilities in the next period which is why credit limits and consumption insurance are barely affected by changes in signal precision.

3 Numerical algorithm for the *ESC* model

With wages normalized to unity and asset prices pinned down by the no-arbitrage condition as stated in the equilibrium condition in the main text, households' optimization problem can be recursively written as

$$V(a,s) \quad = \quad \max_{c,\{a'\}} \left\{ (1-\beta) u[c(a,s)] + \beta \sum_{s'} \pi(s'|s) V'[a'(a,s;s'),s'] \right\}$$

subject to a budget and a borrowing constraint

$$c + \sum_{s'} \frac{\pi(s'|s)a'(a,s;s')}{R} \quad \leq \quad y + a \tag{11}$$

$$a'(a,s;s') \quad \geq \quad A(s'), \quad \forall s'. \tag{12}$$

The borrowing limits satisfy the following equations

$$U^{Aut}(s') = V'[A(s'),s'], \quad \forall s'. \tag{13}$$

The first-order conditions are

$$u'[c(a,s)](1-\beta) = \lambda = V_a(a,y) \tag{14}$$

$$\beta V_a'[a'(a,s;s'),s'] \leq \frac{u'[c(a,s)](1-\beta)}{R}, \quad \forall s', \tag{15}$$

where $V_a'[a'(a,s;s'),s']$ denotes the derivative of the value function with respect to $a'(a,s;s')$. Consider N income states such that $s \in S = (s_1, s_2, ...s_{N^2})$. Consider a grid for a. Start with a guess of the value function V_0, for the derivative of the value function with respect to assets $V_{a,0}$ and a discount factor β. From the guess of the value function, back out the state-dependent borrowing limits $A_0(s')$ from (13).

1. For each pair a, s, solve for the policy functions $c_0(a,s), \{a_0'(a,s;s')\}$ using the N^2+1 first-order conditions (15) and (11). Start with the strict equality for all s' and solve. Check borrowing constraints. If not satisfied in some state s', set $a_0'(a,s;s') = A_0(s')$ and solve again for $c_0(a,s)$ and the remaining $a_0'(a,s;s')$ until no borrowing constraint is violated.

2. Update the derivative of the value function with respect to a using the envelope condition and the policy function for consumption

$$V_{a,1}(a,s) = u'[c_0(a,s)](1-\beta)$$

3. Update the value function according to the Bellman equation to receive V_1

$$V_1(a,s) = (1-\beta)u[c_0(a,s)] + \beta \sum_{s'} \pi(s'|s) V_0[a_0'(a,s;s'),s']$$

4. Continue until convergence in the policy functions, the derivative of the value function and in the value function $V_n(a,s) = V_{n+1}(a,s) = V(a,s)$ is achieved.

5. Then update the borrowing limits solving the following equation for A_1

$$V[A_1(s'),s)] = U^{aut}(s').$$

6. Continue until convergence in the policy functions, in the value function (and its derivative) and in the borrowing limits is achieved.

In the next step, use the policy functions $\{a'(a,s;s')\}$ and transition probabilities $\pi(s'|s)$ to define an operator T that maps the current probability measures for assets and the income-signal state into future measures. In the next step, compute the unique fixed point of the operator T and denote it by $\Phi_{a,s}$, the invariant distribution of assets and income-signal states. Using the invariant distribution compute the excess demand

$$d_K(\beta) = \int c(a,s) \, \mathrm{d}\, \Phi_{a,s} + K' - K(1-\delta) - AF(L,K).$$

Table 2: Risk-sharing, insurance and advance information with alternative estimates

Risk-sharing ratio, RS			Regression coefficient, $\beta_{\Delta y}$		
$\tilde{\kappa} = 0.00$ (0.07)	$\tilde{\kappa} = 0.138$ (0.46)	**Data**	$\tilde{\kappa} = 0$ (0.07)	$\tilde{\kappa} = 0.123$ (0.44)	**Data**
0.94	0.60	**0.47**	0.01	0.16	**0.16**

Notes: ESC model. Risk-sharing ratio and regression coefficient with alternative estimates (data) and in the model for different values of $\tilde{\kappa}$. Values for κ in parentheses. Uninformative signals, $\tilde{\kappa} = 0.00$ or $\kappa = 0.07$ and $N = 14$. Informative signals, $N = 6$.

and check whether it is satisfied. If not, decrease β if $d_K(\beta)$ is in surplus and increase β if it is in deficit, and go back to Step 1. We use a Ridder algorithm until convergence on the discount factor is achieved and excess demand equals zero.

4 Robustness exercises

In this section, we present further robustness exercises on the quantified amount of advance information. First, we address the issue of a potential measurement error in CEX data which would in a lower estimate for RS than the baseline estimate in the main text. In a similar spirit, we also consider an alternative estimate for the regression coefficient $\beta_{\Delta y} = 0.11$ as suggested by Gervais and Klein (2010). Finally, we study how advance information affects the sensitivity of consumption growth with respect to income increases and decreases separately.

Lower risk-sharing ratio Aguiar and Bils (2015) and Attanasio, Hurst, and Pistaferri (2012) argue that the consumption expenditures reported in the CEX Interview Survey may suffer from non-classical measurement error, resulting in biased estimates of cross-sectional consumption inequality measures. In particular, Aguiar and Bils (2015) find that consumption inequality (measured as the cross-sectional variance of logged consumption) has not increased by less than income inequality (measured as the cross-sectional variance of logged income) but moved hand-in-hand with income inequality from 1980-2003. We find that quantified uncertainty gap is not very sensitive with respect to a potentially noisy estimate for consumption inequality. Even if the correct insurance ratio was different than the number computed directly from the CEX, the identified uncertainty gap would only be mildly affected. Suppose that consumption inequality has mirrored income inequality between 1980 and 2003 which results in an risk-sharing ratio of 0.47 instead of the 0.60 we report in the paper.[2] As can be seen in the second column of Table 2, the model can capture the modified risk-sharing ratio with a slightly higher value for the size of the uncertainty gap, $\tilde{\kappa} = 0.138$ instead of 0.124 as in the baseline calibration exercise.

Less insurance Gervais and Klein (2010) argue that the standard estimator $\beta_{\Delta y} = 0.11$ tends to overstate the degree of insurance in CEX data, and propose an alternative estimator. Using the same data as we do but employing the procedure proposed by Gervais and Klein

[2] Assume that consumption inequality increases with the same rate as income inequality (0.5012) such that consumption inequality is 0.1938 instead of 0.1462. Thus, the insurance ratio is 0.47 which requires $\tilde{\kappa} = 0.1376$ to capture the modified risk-sharing ratio.

Table 3: Conditional income and consumption growth regression: *ESC* model

	$\tilde{\kappa} = 0.00$ (0.07)	$\tilde{\kappa}_2 = 0.116$ (0.43)	**Data**
$\beta_{\Delta y_t \mid \Delta y_t \geq 0}$	0.02	0.19	**0.08**
	[0.00]	[0.00]	[**0.00**]
$\beta_{\Delta y_t \mid \Delta y_t < 0}$	7.8×10^{-4}	0.03	**0.09**
	[0.00]	[0.00]	[**0.00**]

Notes: ESC- model. Regression coefficients and their p-values for the regression equation $\Delta c_{it} = \beta_0 + \beta \Delta y_{it} + \epsilon_{it}$, $\beta \in \{\beta_{\Delta y_t|\Delta y_t \geq 0}, \beta_{\Delta y_t|\Delta y_t<0}\}$, $\beta_{\Delta y_t|\Delta y_t \geq 0}$ ($\beta_{\Delta y_t|\Delta y_t<0}$) is the regression coefficient conditional on income increases (decreases) in the model. For the data, the corresponding regression equation in the main text is modified to condition on income increases and decreases. $p-$values are reported in square brackets. Values for κ in parentheses. Uninformative signals, $\tilde{\kappa} = 0.00$, $\kappa = 0.07$ and $N = 14$. Informative signals, $N = 6$.

(2010), Broer (2013) estimates a value of $\hat{\beta}_{\Delta y} = 0.16$ (see Row 7, Column 6 of Table 3 on page 132), implying that consumption growth reacts more sensitively to income changes than in our baseline estimation. As displayed in the fifth column of Table 2, alternatively matching this value of the regression coefficient, we need more precise signals: $\tilde{\kappa} = 0.123$ instead of $\tilde{\kappa} = 0.116$, a value that is very close to $\tilde{\kappa} = 0.124$ which yields an insurance ratio of 0.60 in the model.

Insurance with respect to income increases and decreases In the main text, we compute the regression coefficient $\beta_{\Delta y}$ as an average over the consumption changes to all changes in income. An alternative is to condition the sensitivity of consumption changes on whether income is increasing or strictly decreasing between $t-1$ and t. In Table 3, we display the corresponding estimation results using the model with and without advance information and in the data. In the data, the ratio of regression coefficients with respect to income increases and decreases, $\beta_{\Delta y_t|\Delta y_t \geq 0}/\beta_{\Delta y_t|\Delta y_t<0}$, is with a ratio of close to one almost symmetric. As can be seen in the first two columns, the *ESC*-model produces regression coefficients that imply an asymmetry: as a feature of optimal insurance with complete markets, consumption reacts more sensitively to income increases than decreases, implying that negative income shocks are better smoothed than income positive shocks. Advance information reduces the asymmetry from a ratio of 26.64 without advance information to 6.33 with signals characterized by $\tilde{\kappa} = 0.116$. The improvement with advance information is due to the fact that negative income shocks are no longer perfectly insured with advance information. Advance information therefore brings the model closer to the data but cannot completely reconcile the model with the data.

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