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Public consumption over the business cycle

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What fraction of the business cycle volatility of government purchases is accounted for as endogenous reactions to overall macroeconomic conditions? We answer this question in the framework of a neoclassical representative household model where the provision of a public consumption good is decided upon endogenously and in a time-consistent fashion. A simple version of such a model with aggregate productivity as the sole driving force fails to match important features of the business cycle dynamics of public consumption, which comes out as not as volatile and persistent as in the data and too synchronized with the cycle. We add implementation lags and implementation costs in the budgeting process to the model, plus taste shocks for public consumption relative to private consumption, and achieve a better fit to the data. All these ingredients are essential to improve the fit. In our baseline specification *50%* of the variance of public consumption is driven by aggregate productivity shocks.

Keywords. Public consumption, aggregate productivity shocks, business cycles, implementation lags, implementation costs, taste shocks, time-consistent public policy.

JEL classification. E30, E32, E60, E62, H30.

1. Introduction

Standard business cycle analysis often treats government purchases as an *exogenous* stochastic process. As such they appear in at least three different strands of the literature: as a wedge and potential driving force of aggregate fluctuations (see Baxter and King (1993), Chari, Kehoe, and McGrattan (2007), or Leeper, Plante, and Traum (2010), for instance), in the empirical literature on the sign and magnitude of the government spending multiplier as a source of an exogenous shock to be identified (see Shapiro

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and Ramey (1998), Blanchard and Perotti (2002), Mountford and Uhlig (2009), or Ramey (2011), for example), and in the optimal fiscal policy literature (see Chari and Kehoe (1999), and Kocherlakota (2010) for an overview), where there is an exogenous stream of government purchases that needs to be financed by either taxes or debt.

In this paper, we reverse the perspective and ask, "Once we allow for an *endogenous* public good provision, what fraction of the business cycle fluctuations of government purchases is accounted for as endogenous reactions to overall macroeconomic conditions? Additionally, how much volatility is generated through shocks related to the provision of public goods?"

To answer these questions, we start by documenting the business cycle properties of public consumption. We define public consumption as the counterpart of private consumption within government purchases, namely "government expenditures on consumption and investment goods," as stipulated in the National Income and Product Accounts (NIPA). More specifically, the annual percentage volatility of aggregate state and local government consumption is with roughly 1.80%, almost as high as that of aggregate gross domestic product (GDP), 1.90%.[1] Its persistence is 0.77 and with 0.24, it has the lowest contemporaneous correlation coefficient with aggregate output, lower than for any other component of domestic aggregate expenditures. Unlike private consumption, its dynamic correlations with 1- and 2-year lagged GDP are with 0.39 and 0.38, respectively, higher than its contemporaneous correlation with GDP.

We then draw on previous work by Klein, Krusell, and Rios-Rull (2008; KKR henceforth) for our quantitative analysis. The KKR framework is a natural starting point for this analysis, because it features a standard neoclassical growth model and adds to it time-consistent public policy.[2] The model has a government that cannot commit ex ante to a path of future public consumption, but takes into account this path and how it depends on current decisions. The solution concept for the game between successive governments is the Markov-perfect equilibrium. Public consumption is financed by linear income taxes. We abstract from government debt and transfers. We discipline the exercise by requiring that the model fit as closely as possible the business cycle features of public consumption described above.

As a first step, we add aggregate productivity shocks as the sole aggregate driving force to the KKR framework. Our first result is that such a model compared to the data falls short in terms of volatility (1.14% in the model versus 1.80% in the data) and in terms of persistence (0.49 in the model versus 0.77 in the data), and it makes government consumption almost perfectly and contemporaneously correlated with the cycle.

Motivated by the dynamic correlation pattern of government consumption in the data, we add an implementation lag to the physical environment: today's government can only decide about public consumption tomorrow and tomorrow's government is

[1]We use annual NIPA data from 1960–2006. Details on other aggregates of public consumption, which include the federal level, can be found in Section 2.

[2]In the words of Kocherlakota (2010), "These literatures [on time-consistency and dynamic political economy] examine the properties of equilibrium outcomes of particular dynamic games. Hence, they are trying to model *actual* behavior of governments" (emphasis in the original).

bound by this decision. Implementation lags are a realistic feature of the budgeting process given the numerous bureaucracies involved with government expenditures. This helps us push the peak correlation of public consumption and output away from contemporaneous, but still leaves us with too high a dynamic correlation and too low persistence.

We then include a taste shock for public consumption (relative to private consumption) in the flow utility of the representative household, which leads to a decoupling of economic aggregates and government consumption. While we use a preference shock, we think of this taste shock as a way to capture more generally fluctuations in the political system directly related to the provision of public goods. On its own, such a shock does not lead to sizable output fluctuations or realistic business cycles. Moreover, this second shock reduces the persistence of government consumption further compared to the data.

We remedy this, finally, by introducing implementation costs (in addition to the implementation lags). We thus assume that it is costly for governments to deviate too much from previous budgets. Implementation lags and costs are modeled similarly to, respectively, time-to-build and convex adjustment costs for capital in standard macroeconomic models.

Our second result is that within the class of models we are studying, the model with two aggregate shocks and two implementation frictions (in addition to the "no-commitment" friction) provides the best joint fit to all three dimensions of the business cycle dynamics of government purchases: volatility, persistence, and dynamic comovement.

Our final result is the answer to our original research question: in our baseline specification and using the best fitting model within this model class, 50% of the fluctuations of public consumption are explained by endogenous reactions to macroeconomic conditions.

Related literature

In addition to the general link of our paper to the literature on time-consistency issues in public decision making, our paper in particular relates to three strands of the literature on the cyclical dynamics of fiscal policies.[3]

First, our paper belongs to a literature on the endogenous responses of government spending to economic shocks. A number of authors have studied the amplification and propagation mechanisms of public expenditure in response to aggregate shocks, including total factor productivity (TFP) shocks (Ambler and Paquet (1996), Barseghyan, Battaglini, and Coate (2010), Debortoli and Nunes (2010), Azzimonti and Talbert (2011), Bachmann and Bai (2013)), preference shocks (Battaglini and Coate (2008), Azzimonti, Battaglini, and Coate (2010), Yared (2010)), commitment shocks (Debortoli and Nunes

[3]A complementary literature on endogenous public policy focused on deterministic policy dynamics: in addition to KKR, see Krusell, Quadrini, and Rios-Rull (1997), Krusell and Rios-Rull (1999), Hassler et al. (2003, 2005), Corbae, D'Erasmo, and Kuruscu (2009), Martin (2010), Azzimonti (2011), Bai and Lagunoff (2011), and Song, Storesletten, and Zilibotti (2012).

(2010, 2013)), and political uncertainty shocks (Woo (2005), Azzimonti and Talbert (2011)). In particular, Azzimonti and Talbert (2011) share common elements with our paper, but with a focus on the effects of TFP and political uncertainty on emerging-market consumption volatility. Like the preference shocks in this paper, the political uncertainty in their model increases the volatility of public consumption and dampens the contemporaneous co-movement between government spending and GDP.

Second, our paper also relates to the research on endogenous movements of capital and labor tax rates over the business cycle: see Chari, Christiano, and Kehoe (1994), Stockman (2001), and Klein and Rios-Rull (2003) as well as Feng (2012). These papers emphasize the distinct responses of capital and labor tax rates to exogenous government spending shocks. Complementary to these papers, we focus on the endogenous movement of government spending as a result of other aggregate shocks.

Finally, this paper relates to a literature on the business cycle patterns of government expenditures in emerging market economies: see Alesina, Campante, and Tabellini (2008), Parmaksiz (2010), Ilzetzki (2011), and Azzimonti and Talbert (2011). This research focuses on the role of sovereign borrowing constraints as well as the role of financial and political frictions for the excess volatility and procyclicality of fiscal policies in emerging markets.

The remainder of the paper is organized as follows. The next section documents the business cycle facts for government consumption. Section 3 sets up the model and discusses its computation and calibration. Section 4 presents the results and explains in detail how each of the model features contributes to fitting the model to the observed dynamics of public consumption. A final section concludes. Details are relegated to various appendices.

2. Facts

Table 1 shows the business cycle moments for state and local government consumption (GSLC), our baseline government purchases aggregate, as well as other subaggregates of

Table 1. Business cycle facts: government consumption.

Moment	GSLC	GNDC	GND	GC	G	CNDS
std$(\cdot)$	1.796%	1.520%	1.871%	2.405%	2.813%	1.106%
rho$(\cdot)$	0.774	0.704	0.741	0.781	0.794	0.629
correl$(\cdot, Y)$	0.237	0.188	0.468	0.255	0.347	0.853
correl$(\cdot, Y_{-1})$	0.389	0.341	0.577	0.434	0.514	0.510
correl$(\cdot, Y_{-2})$	0.379	0.404	0.498	0.509	0.511	0.099
correl$(\cdot$, CNDS$)$	0.534	0.415	0.526	0.350	0.391	—
correl$(\cdot$, CNDS$_{-1})$	0.582	0.510	0.600	0.428	0.477	—
correl$(\cdot$, CNDS$_{-2})$	0.484	0.549	0.510	0.454	0.432	—

Note: The data source is the Bureau of Economic Analysis (BEA) (NIPA data). All variables are annual and the sample goes from 1960 to 2006. They are deflated by their corresponding deflators, and logged and filtered with a Hodrick–Prescott filter with smoothing parameter 100. GSLC stands for state and local government consumption. GNDC denotes total nondefense consumption, GND denotes total nondefense purchases, GC denotes total government consumption, and G is total government purchases; std$(\cdot)$ denotes the time-series volatility of an aggregate variable in percent, rho$(\cdot)$ denotes its first-order autocorrelation, correl$(\cdot, Y)$ denotes the contemporaneous correlation with aggregate GDP, and correl$(\cdot, Y_{-1})$ and correl$(\cdot, Y_{-2})$ denote the correlation with aggregate 1- and 2-year lagged GDP, respectively. CNDS stands for nondurable and services consumption.

total government purchases. All variables are annual, and logged and detrended with a Hodrick–Prescott (HP) filter with a smoothing parameter of 100. We make the following observations.

1. GSLC is more volatile than private consumption expenditures, measured as spending on nondurables and services, and almost as volatile as GDP (1.897%).
2. GSLC is persistent, at least as persistent as GDP (0.541).
3. GSLC is procyclical, dynamically more so than contemporaneously.

State and local government consumption belongs by definition to the nondefense category, which is a plausible candidate for endogenous expenditures. The structural vector autoregressions literature often takes the same view and uses military purchases to identify exogenous government spending shocks. Focusing on the state and local level also allows us to abstract from government debt, which would complicate the model and the computation considerably. Furthermore, GSLC is roughly 10% of GDP and slightly under 50% of total government purchases, which makes it the largest individual category at this level of aggregation.

In any event, Table 1 also shows that total nondefense government consumption (GNDC), which includes federal consumption expenditures, has similar business cycle properties to GSLC. Persistence is above 0.7 in all subaggregates. In addition, the dynamic correlation pattern of state and local government consumption can also be found in other aggregates, such as nondefense purchases, total public consumption, and total government purchases. We view this as at least suggestive that the causes of the business cycle of government purchases should be sought in aggregate factors.

Tables 10 and 11 in Appendix A provide some robustness analyses. Table 10, for instance, shows that the broad patterns we see in GSLC, in particular persistence and an increasing dynamic correlogram, also hold true for a functional disaggregation of government purchases, which suggests that our findings are not driven by composition effects. Table 11, using an HP filter smoothing parameter of 6.25 (see Ravn and Uhlig (2002)) shows that our results are broadly robust to using different business cycle filters.[4] Finally, Figures 2–4 in the same appendix show, using data from the Annual Survey of State Government Finances, that the dynamic correlation pattern for aggregate state and local government consumption with GDP also holds for most U.S. states individually.

The evidence taken together leads us to treat the three properties of GSLC from the beginning of this section as *stylized business cycle facts*. They are also suggestive of some of the model ingredients we use in the quantitative exercise that follows. The fact that the dynamic correlogram between public consumption and output/private consumption is tilted toward public consumption lagging the cycle suggests implementation lags. We also show that without a second shock, a representative agent model overshoots the level of the correlogram (see Bachmann and Bai (2013), for an alternative story in a heterogeneous agent framework). Finally, the persistence numbers suggest the budget implementation costs we use.

[4] There are, however, quantitative differences: the level of the correlogram, not its overall shape, is lower with more flexible trends, as are the persistence numbers and the volatilities.

3. The model

The environment is a neoclassical representative household one-sector growth model with valued public consumption. The government finances the provision of the public good with a flat rate income tax and adheres to a balanced budget rule, which for government consumption approximates well most U.S. states' constitutions. The government cannot commit ex ante to future public policy. Government consumption is chosen to maximize the welfare of the representative household. The equilibrium is subject to a time-consistency requirement.

3.1 *The economic environment*

The economy is populated by a unit mass continuum of infinitely lived identical households. In each period, the household is endowed with $\tilde{l}$ units of time. It values private consumption, c, leisure, $\tilde{l}-l$, and government consumption, G, according to the felicity function

$$u(c, l, G) = \eta\big(\theta \log(c) + (1-\theta)\log(G)\big) + (1-\eta)\log(\tilde{l}-l). \tag{1}$$

Lifetime utility follows the standard expected utility form with a discount factor β. The parameter θ that governs the relative preferences for private consumption versus public consumption is assumed to be time-varying. We interpret this taste shock as a shock that directly affects the provision of consumption in the form of private versus public goods, but otherwise does not generate realistic economic business cycle fluctuations. For example, this θ shock does not cause any sizable output fluctuations. Specifically, we assume that $\theta = \bar{\theta}\hat{\theta}$, where $\hat{\theta}$ follows a two-state symmetric Markov chain with support $[1-\epsilon_\theta, 1+\epsilon_\theta]$ and transition matrix $\left(\begin{smallmatrix} \rho_\theta & 1-\rho_\theta \\ 1-\rho_\theta & \rho_\theta \end{smallmatrix}\right)$, ϵ_θ governs the volatility of this process, and ρ_θ governs its persistence.

Notice that with a time-varying θ, we implicitly assume here that the relative taste shock is primarily between private and public *consumption* with only an indirect leisure effect. We want to highlight the time-varying tastes in the population between private and public provision of physical commodities and use this formulation as our baseline case.[5]

The household owns capital, k, and rents it out in a perfectly competitive market. Capital depreciates at rate δ. The budget constraint of the household is given by

$$c + k' = (1-\delta)k + (1-\tau)(wl + rk), \tag{2}$$

where k' is the capital carried over to the next period, τ is the flat income tax rate, w is the real wage, and r is the rental rate for capital. The capital k' is restricted to lie in $[0, +\infty)$.

Aggregate output, Y, is produced by a representative firm according to an aggregate Cobb–Douglas production function $Y = zK^\alpha L^{1-\alpha}$, where K and L are the aggregate capital stock and the aggregate labor input, respectively, and z denotes aggregate

[5]With three commodities in the felicity function there is another formulation where the taste shock is between public consumption and the private bundle *including* leisure. We explore this specification, as well as one with inelastic labor supply, in Section 4.3: $u(c, l, G) = \theta(\eta \log(c) + (1-\eta)\log(\tilde{l}-l)) + (1-\theta)\log(G)$.

productivity and is the baseline source of aggregate uncertainty in this economy that generates realistic economic business cycles. Its natural logarithm evolves according to a Gaussian AR(1) process. The firm rents capital and hires labor from the household at the rental rate r and the wage rate w. Competitive factor markets guarantee the usual factor pricing conditions $w(K, L, z) = (1-\alpha)(K/L)^{\alpha}$ and $r(K, L, z) = \alpha z(K/L)^{\alpha-1}$.

Government consumption is decided one period ahead. We assume that the current government is legally bound by this decision and, in this sense, there is a one-period-ahead commitment. This feature captures implementation lags in the budget process. In addition, the budget authority pays a quadratic adjustment cost for changing the next period's government consumption. This is meant to capture budget planning costs, where budget items can only be changed in small steps, because of vested political interests behind them.[6] Both government consumption of the current period and the adjustment costs are financed by the flat tax on current income through a balanced budget requirement:

$$\tau Y = G + \frac{\Omega}{2}\left(G' - G\right)^2. \tag{3}$$

The flat income tax rate is thus implicitly defined as a function of (K, L, z, G, G'):

$$\tau\left(K, L, z, G, G'\right) = \frac{G + \frac{\Omega}{2}(G' - G)^2}{zK^{\alpha}L^{1-\alpha}}. \tag{4}$$

Aggregate output is used for private and public consumption, plus budget adjustment costs, as well as private investment:

$$C + G + \frac{\Omega}{2}\left(G' - G\right)^2 + K' = (1-\delta)K + zK^{\alpha}L^{1-\alpha}. \tag{5}$$

3.2 *Equilibrium with endogenous public policy*

Tomorrow's government consumption is chosen to maximize the welfare of the representative household today. When deciding tomorrow's G, the government does not have commitment power into the future beyond tomorrow. Without a commitment device, it is well known that the commitment equilibrium in our environment is typically not time-consistent. Time consistency thus requires imposing a subgame-perfect restriction with successive governments and the households as game players. Following Krusell and Rios-Rull (1999) and KKR, we focus on a subclass of subgame-perfect equilibrium with Markov strategies, that is, Markov-perfect equilibrium (MPE). The formal definition follows.

Definition 1. A Markov-perfect equilibrium for the economy is a set of functions, including a government policy function $G' = \Psi(K, G, z, \theta)$, a transition function $K' =$

[6]The literature, most recently Fernandez-Villaverde et al. (2012), has documented that estimated reduced-form fiscal policy rules usually find the autoregressive coefficient on fiscal policy variables to be close to 1, which comports with our findings that budget implementation costs are important, using a more structural approach.

$H(K, G, z, \theta, G')$, an aggregate labor supply function $L(K, G, z, \theta, G'; \Psi, H)$, a tax function $\tau(K, L, z, G, G'; \Psi, H)$, an equilibrium continuation value function $v(k, K, G, z, \theta; \Psi, H)$, a best-response value function $J(k, K, G, z, \theta, G'; \Psi, H)$, and a best-response decision rule $k' = h(k, K, G, z, \theta, G'; \Psi, H)$ and $l = l(k, K, G, z, \theta, G'; \Psi, H)$, such that the following statements hold:

(a) For any given G', the value functions and decision rules solve the household problem

$$\begin{aligned} & J(k, K, G, z, \theta, G'; \Psi, H) \\ & \quad = \max_{\{c,l,k'\}} \{u(c, l, G) + \beta E[v(k', K', G', z', \theta'; \Psi, H)|z, \theta]\} \\ & \quad \text{s.t.} \quad c \geq 0, \qquad k' \geq 0, \qquad 0 \leq l \leq \tilde{l} \\ & \qquad c + k' = (1-\delta)k + (1 - \tau(K, L, z, G, G'))(w(K, L, z)l + r(K, L, z)k), \\ & \qquad K' = H(K, G, z, \theta, G'), \\ & \qquad L = L(K, G, z, \theta, G'; \Psi, H). \end{aligned}$$

In addition, $v(k, K, G, z, \theta; \Psi, H) = J(k, K, G, z, \theta, \Psi(K, G, z, \theta); \Psi, H)$.

(b) We have $H(K, G, z, \theta, G') = h(K, K, G, z, \theta, G'; \Psi, H)$ and $L(K, G, z, \theta, G'; \Psi, H) = l(K, K, G, z, \theta, G'; \Psi, H)$.

(c) The function $\Psi(K, G, z, \theta)$ maximizes the welfare of the representative household on the equilibrium path, that is,

$$\Psi(K, G, z, \theta) = \arg\max_{G'} \{J(K, K, G, z, \theta, G'; \Psi, H)\}. \tag{6}$$

(d) The government budget constraint is satisfied: $\tau(K, L, z, G, G'; \Psi, H) = \frac{G + (\Omega/2)(G'-G)^2}{zK^\alpha L(K,G,z,\theta,G';\Psi,H)^{1-\alpha}}$.

The first part of the equilibrium definition says that the household decision rules should be the best response to an arbitrary decision on G' when the future follows the equilibrium path, a so-called one-shot deviation best response. The symbol J denotes the value function that corresponds to these optimal household decisions. In addition, the best-response value function should coincide with the equilibrium continuation value function when evaluated at the equilibrium policy $G' = \Psi(K, G, z, \theta)$.

The second part of the equilibrium definition requires that the evolution of the aggregate capital stock and labor supply are both generated by the household's best responses. This reflects rational expectations on the household side for both the on- and off-equilibrium path. On the equilibrium path, this requirement reduces to the familiar consistency restriction in a recursive competitive equilibrium. The third and fourth parts specify the constitutional rule for the choice of public consumption tomorrow.[7]

[7]Our equilibrium definition can be written in an alternative form (henceforth Definition B) just as in Appendix B of KKR (p. 806), which was shown there to be equivalent to a more compact formulation pre-

3.3 *Computation and calibration*

We use numerical methods to characterize and analyze the Markov-perfect equilibrium of the specified economy. As already intimated in the equilibrium definition, we iterate on the capital transition function and policy rule (H, Ψ) until a fixed point is reached. The fixed point of H takes the form

$$\begin{aligned} \log K' = {} & a_0(z,\theta) + a_1(z,\theta)\log K + a_2(z,\theta)\log G + a_3(z,\theta)\log G' \\ & + a_4(z,\theta)\big(\log G'\big)^2 + a_5(z,\theta)\big(\log G'\big)^3 + a_6(z,\theta)\log G \log G' \end{aligned} \tag{7}$$

and that of Ψ takes the form

$$\log G' = b_0(z,\theta) + b_1(z,\theta)\log K + b_2(z,\theta)\log G. \tag{8}$$

Notice that these functions depend, through the coefficients $a_i(\cdot,\cdot)$ and $b_i(\cdot,\cdot)$, on the level of aggregate productivity and the taste for private versus public consumption. As for the functional form in (7), we started with a simple log-linear rule instead of (7), but found the $R2$ to be somewhat low, at least for some specifications of the model. After some experimentation, (7) turned out to be a good compromise between numerical stability and accuracy. Notice that H has to have good predictive power not only on-equilibrium, but also for a grid of off-equilibrium proposals for G'. The average $R2$ over the discrete number of aggregate states improves from 0.9371 to 0.9998 for the baseline model when we add nonlinear terms.[8]

We set the output elasticity of capital $\alpha = 0.36$ and set the labor scale $\tilde{l} = 1$. For other parameters, the model is calibrated to match important features of the U.S. economy from 1960 to 2006. Annual data on government consumption correspond closely to the yearly nature of government budgeting and, therefore, we calibrate our model to this frequency. This choice implies three parameter selections: first, the depreciation rate, δ, is set to 0.1; second, the discount rate, β, is fixed at 0.96. Third, following Tauchen (1986), we model aggregate productivity, z, as a five-state Markov chain that approximates a Gaussian log AR(1) process with an autocorrelation coefficient of 0.8145 (i.e., 0.95 to the power of four; see Cooley and Prescott (1995)) and, in the baseline calibration, conditional standard deviation of 0.0123. This standard deviation is chosen to make our models approximately match the annual percentage standard deviation of GDP in the data, 1.90%. This paper is not concerned with explaining output volatility from a measured exogenous shock series, as the Real Business Cycle (RBC) tradition, which uses fluctu-

sented in Section 2.3 of the main body of their paper. The bottom line is that KKR used an Euler equation formulation, whereas we use a value function formulation, as in Krusell and Rios-Rull (1999). In particular, the combination of parts (a) and (b) of our definition is equivalent to part 2 (the household Euler equation) and part 3 (the continuation value) of Definition B, and part (c) corresponds to part 1, where H in our formulation relates to $\widetilde{H}$ in Definition B. Consequently, our definition is equivalent to both definitions in KKR, assuming that their Euler equation is also sufficient for optimality.

[8]See Appendix B for an outline of the algorithm, the coefficients of the equilibrium law of motion, and the government policy function for the baseline case in Tables 12 and 13, and (in Table 14) the comparison in fit between the baseline version where we use (7) and the version where we use only the terms until $a_3(z,\theta)\log G'$ for the parameterization of H.

ations in the Solow residual to generate a large part of observed output fluctuations. Rather, this paper is about explaining government consumption dynamics (and other components of aggregate demand), given the correct output fluctuations.

The two parameters in the felicity function are calibrated as follows: $\bar{\theta} = 0.8512$, the average love-of-private-consumption parameter, is picked to match the time-averaged $\frac{G}{Y}$ ratio based on aggregate state and local government consumption, that is, roughly 10.2%[9]; η, the parameter that specifies the relative weight between the private–public-consumption composite and leisure, is chosen to make average labor hours 0.33, that is, $\eta = 0.4013$, in the baseline case.

Three nonstandard parameters remain to be calibrated: ρ_θ, ϵ_θ, and Ω. We fix ρ_θ at 0.75, which means that a given taste for government consumption remains operative for 4 years on average. The parameters ϵ_θ and Ω are chosen to minimize a weighted quadratic form in the following summary statistics for the dynamics of public and private consumption: the standard deviations and first-order autocorrelations of public and private consumption, the contemporaneous and 1- and 2-year lagged correlations of public consumption with GDP and private consumption, and the contemporaneous and 1-year lagged correlations between private consumption and GDP. These statistics (numbers can be found in Table 1) summarize the joint business cycle dynamics of public and private consumption as well as GDP.

Specifically, let M be the collection of the aforementioned business cycle moments in the data and let $\hat{M}_i$ be the same collection of moments from the ith simulation of the model. Then we minimize: $\|(M - \frac{1}{190}\sum_{i=1}^{190}\hat{M}_i)/W\|$, where W denotes the conforming collection of standard deviations of the 12 time-series moments in the data (see Table 15 in Appendix B for details) and $\|\cdot\|$ is the Euclidean norm.[10] We use 190 simulations of length 40 to compute the model-based moments.

3.4 *Several benchmarks*

Our baseline model introduces two new frictions, that is, budget implementation lags and budget implementation costs, in addition to distortionary taxes and limited commitment. To isolate the effects of the new frictions, we compare our baseline model with three benchmark models, with increasing degrees of deviation from the first-best allocation. As a starting point, in the frictionless Pareto Model, the social planner, in the presence of exogenous aggregate TFP shocks, chooses the allocation to maximize the utility of the representative household subject to the resource constraint. It is well known that the optimal allocation coincides with a decentralized competitive equilibrium with lump-sum taxation.

[9]To take into account the higher distortion from higher government expenditures that in reality include federal spending, investment spending, transfers, and so on, we also study a calibration where we posit a fixed amount of wasteful government spending that is not decided over, so as to also match the ratio of total government revenues to GDP in the data: 0.287. While the details of the calibration are somewhat different, our basic results do not change under this specification. They are available on request from the authors.

[10]We have also experimented with a mean absolute deviation criterion with similar results.

If the only financing instrument of the government is a distortionary linear income tax, as in the Ramsey taxation framework, the government can still achieve the second-best allocation provided that it has access to a full-commitment technology. In this "Ramsey Model," the government picks the welfare-maximizing competitive equilibrium by choosing the dynamic path of public consumption and tax rates jointly in response to TFP shocks. We provide a formal discussion of this model in Appendix C.

Finally, when we eliminate the ability of the government to commit to any future policy in the Ramsey Model, we arrive at our point of departure, the KKR model subject to standard aggregate TFP shocks, which we call the Simple Model.

4. Results

4.1 *Main results*

Table 2 summarizes two of our three main results. First, a Simple Model with no implementation lags, no implementation costs, and only aggregate productivity shocks can generate a volatility of public consumption (1.14%) that is lower than that observed in the data (1.80%). It delivers lower persistence, 0.49 versus 0.77 in the data, and the wrong correlogram for public consumption. Second, the baseline model with an implementation lag and calibrated implementation costs as well as a taste shock does substantially better in matching the data, especially in terms of persistence and the correlogram, while hardly deteriorating the fit in terms of volatility.[11] It bears pointing out that

Table 2. Baseline result.

Business Cycle Moment	Baseline Model	Simple Model	Ramsey Model	Pareto Model	Data
std(G)	1.087%	1.144%	0.874%	0.804%	1.796%
rho(G)	0.622	0.492	0.572	0.606	0.774
correl(G, Y)	0.105	0.970	0.918	0.888	0.237
correl(G, Y_{-1})	0.559	0.531	0.617	0.645	0.388
correl(G, Y_{-2})	0.546	0.201	0.324	0.369	0.379
correl(G, C)	0.409	0.960	0.930	1.000	0.534
correl(G, C_{-1})	0.449	0.407	0.492	0.606	0.582
correl(G, C_{-2})	0.364	0.018	0.119	0.226	0.484
std(C)	0.979%	0.842%	0.770%	0.804%	1.106%
rho(C)	0.543	0.612	0.657	0.606	0.629
correl(C, Y)	0.824	0.885	0.787	0.888	0.853
correl(C, Y_{-1})	0.605	0.652	0.684	0.645	0.510

Note: The baseline model features both a 1-year implementation lag and implementation costs ($\Omega = 25$), as well as $\epsilon_\theta = 0.006$. The Simple Model has no implementation lags or costs ($\Omega = 0$) and $\epsilon_\theta = 0$, but the government still cannot commit to a future path of government purchases. The Ramsey Model is the same as the Simple Model, but the government has commitment. The Pareto Model is the same as the Ramsey Model except that the government can levy lump-sum taxes. All time series for both actual and model-simulated data are logged and HP(100) filtered. The model-based moments have been computed as the average from 190 simulations of length 40. Public consumption in the data refers to GSLC (state and local government consumption). Private consumption in the data refers to CNDS (nondurable and services consumption). The abbreviation std denotes the standard deviation and rho denotes the first-order autocorrelation of the corresponding time series.

[11]Table 16 in Appendix D shows that the baseline model also does well in matching the same statistics when we replace public and private consumption with their respective ratios over aggregate output. The

in the baseline model, the aggregate budget implementation costs paid are just below 0.01% as a fraction of government purchases and as a fraction of GDP they are just below 0.001%.[12]

The columns "Ramsey Model" and "Pareto Model" compare our baseline model and the Simple Model without commitment to the other two benchmark economies. We will explain the model differences in terms of a trade-off between two well known smoothing motives: smoothing public consumption, which the benevolent government wants just as it desires smooth private consumption, and smoothing taxes in the presence of distortionary taxation, that is, distributing the tax distortion optimally over time.

As expected, in the Pareto Model, public and private consumption have exactly the same time-series properties, the different utility weights only effect their average size, but otherwise they are perfectly correlated, and public consumption is just as smooth as private consumption. This smoothness comes from the standard (private) consumption smoothing force in dynamic decision making, only now it manifests itself in government consumption as well. The addition of the distortionary tax friction in the Ramsey Model introduces a new tax smoothing motive to reduce the dead-weight loss of taxation. Given the lack of access to debt instruments in our model, the government can only smooth taxes over the business cycle through adjustment of the government consumption margin. This makes government consumption more synchronized with GDP and implies a higher volatility as well as lower persistence relative to the Pareto Model, as can be seen in the fourth column of Table 2.

As we take away the ability of the government to commit to future policies, the Simple Model in the third column, the incentives of the government both in terms of tax smoothing and consumption smoothing are altered. On the one hand, the no-commitment government does not fully internalize the current tax distortion on capital accumulation, which reduces the tax smoothing motives. On the other hand, lack of commitment also dampens the dynamic consumption smoothing force, because future public consumption is not directly chosen by the current government. The quantitative results in Table 2 show that the dampened consumption smoothing motive dominates quantitatively so that public consumption displays higher volatility, lower persistence, and stronger co-movement with aggregate output. Neither the Ramsey Model nor the Pareto Model provides an obviously better fit to the data than the Simple Model without commitment, so we use the latter as the point of departure for our analysis.

Table 3 displays our third result, a variance decomposition for public consumption in the baseline model. When we run models with the same parameterization as the baseline model, but shut down, respectively, the taste shocks between private and public consumption and the aggregate productivity shocks, we generate, respectively, 50% and

business cycle moments of other macroeconomic aggregates are standard and similar across the various model specifications. Table 17 in Appendix D shows them for the baseline model.

[12]There is potentially an issue as to whether these budget implementation costs should be included in our measure of G that we compare to the NIPA data. We currently do not include them, which means we implicitly assume that these budget implementation costs are wasted resources inside the government that are not recorded by U.S. NIPA. In practice, it would make no difference. All the statistics we report would not change before the fourth digit, and the calibration of implementation costs and the volatility of the taste shock would not change either.

TABLE 3. Variance decomposition: baseline model.

Contribution of z Shocks	Contribution of θ Shocks	Both
49.58%	40.80%	90.38%

Note: See the footnote to Table 2. The first column displays the fraction of the time-series *variance* of public consumption in the baseline model, when the θ shocks are shut down, but the model is parameterized the same otherwise. The second column shuts down the aggregate productivity shocks. The third column is the sum of these variances.

41% of the variance of public consumption in the baseline model. That these variances do not quite add up to unity is indicative of endogenous interaction effects in the joint response of public consumption to these shocks.

4.2 *Explaining the mechanism*

How do the various elements of the baseline model—implementation lags and implementation costs as well as taste shocks between private and public consumption—contribute toward the model's fit to the data? We address this question in two steps: Table 4 stays within the class of models with implementation lags, but, one step at a time, removes implementation costs and the taste shocks for public consumption from the baseline calibration, keeping all other parameters the same. It also shows how the dynamics of public consumption look in a model with only taste shocks and no aggregate productivity shocks. Table 5 then shows how a model without implementation lags, but the same parameters as the baseline model, fails to reproduce the initially increas-

TABLE 4. The role of implementation costs and taste shocks.

Business Cycle Moment	Baseline Model	No Taste Shock	No Implementation Costs	No Taste Shock, No Implementation Costs	No Productivity Shock
std(G)	1.087%	0.765%	1.850%	1.099%	0.694%
rho(G)	0.622	0.727	0.308	0.498	0.523
correl(G, Y)	0.105	0.140	0.163	0.326	−0.290
correl(G, Y_{-1})	0.559	0.758	0.564	0.965	−0.955
correl(G, Y_{-2})	0.546	0.748	0.321	0.529	−0.564
correl(G, C)	0.409	0.576	0.296	0.653	0.002
correl(G, C_{-1})	0.449	0.963	0.223	0.959	−0.837
correl(G, C_{-2})	0.364	0.753	0.175	0.407	−0.590
std(C)	0.979%	0.911%	0.958%	0.882%	0.379
rho(C)	0.543	0.618	0.531	0.601	0.166
correl(C, Y)	0.824	0.886	0.825	0.892	0.067
correl(C, Y_{-1})	0.605	0.658	0.583	0.642	−0.226

Note: See the footnote to Table 2. The baseline model features both a 1-year implementation lag and implementation costs ($\Omega = 25$), $\epsilon_\theta = 0.006$. The no taste shock model is identical to the baseline model, but sets $\epsilon_\theta = 0$. The no implementation costs model is identical to the baseline model, but sets $\Omega = 0$. The no taste shock–no implementation costs model is a combination of the third and fourth columns. The no productivity shock model is the same as the baseline model, but without aggregate productivity shocks (see the notes to Table 3).

Table 5. The role of implementation lags.

Business Cycle Moment	No Implementation Lag Recalibrated	No Implementation Lag Param. from Baseline	Baseline Model	Data
std(G)	1.103%	1.582%	1.087%	1.796%
rho(G)	0.643	0.527	0.622	0.774
correl(G, Y)	0.362	0.406	0.105	0.237
correl(G, Y_{-1})	0.447	0.394	0.559	0.388
correl(G, Y_{-2})	0.373	0.263	0.546	0.379
correl(G, C)	0.333	0.224	0.409	0.534
correl(G, C_{-1})	0.336	0.227	0.449	0.582
correl(G, C_{-2})	0.262	0.165	0.364	0.484
std(C)	0.954%	0.935%	0.979%	1.106%
rho(C)	0.568	0.551	0.543	0.629
correl(C, Y)	0.824	0.832	0.824	0.853
correl(C, Y_{-1})	0.628	0.608	0.605	0.510

Note: See the footnote to Table 2. The second column shows the results of a model where public consumption is decided on contemporaneously, but implementation costs and the volatility of the relative taste shock between private and public consumption have been calibrated to minimize the same quadratic form as the baseline model: $\Omega = 45$, $\epsilon_\theta = 0.005$. The third column shows the results of a model where public consumption is decided on contemporaneously, but the implementation costs parameter and the volatility of the relative taste shock are set equal to those in the baseline model: $\Omega = 25$, $\epsilon_\theta = 0.006$.

ing correlogram between public consumption and output/private consumption in the data.

The last column of Table 4 shows that taste shock alone would lead to very counterfactual dynamics of public consumption, too little volatility, too little persistence, and negative co-movement with the business cycle. This reiterates in a stark, qualitative sense the result from Table 3 that aggregate productivity shocks are important for our understanding of government consumption fluctuations.

Next, starting from the fifth column in Table 4, we see that a model with no implementation costs and only aggregate productivity shocks delivers too little volatility and persistence compared to the data, and overstates the level of the dynamic correlation between public consumption and lagged private consumption/output. Introducing the taste shocks (fourth column) into the economy improves the dynamic correlation pattern and volatility, but worsens the persistence problem. This means that had we focused only on fitting the model to the persistence of government consumption, taste shocks would play no role. But this would have been at the expense of the model volatility of government consumption vis-à-vis the data, and with insufficient dampening of the dynamic correlogram between public consumption and output as well as private consumption.

Conversely, introducing budget implementation costs only (third column) helps with persistence and, somewhat, the oversynchronization issue between public consumption and the other macroeconomic aggregates, but exacerbates the shortfall of volatility. In other words, had we focused mainly on the volatility of public consumption and the dynamic correlogram, taste shocks would have been the only addition to the model in the fifth column, no implementation costs. In fact, when we decompose, just as in Table 3, the variance of public consumption in a model with both productivity and taste shocks, but no implementation costs, we find that taste shocks now contribute 63% to the variance of public consumption in the model with both shocks, whereas productivity shocks contribute only 35% of the variance.

Combining both implementation costs and taste shocks leads to a model that improves the fit to the data in terms of persistence and the dynamic correlations without worsening the fit in terms of the volatility of public consumption.

The fact that the "simpler" model in the fifth column and the baseline model in the second column have roughly the same volatility of public consumption, but the latter improves on the former in terms of persistence and dynamic correlations, shows that within the class of models studied, *both* additional features—implementation costs and taste shocks—are required by the data. Starting from the simpler model in the fifth column, the additional shock that affects public consumption will lead to larger volatility, but less persistence, whereas implementation costs will lead to insufficient volatility of public consumption, but higher persistence. This means that the physical environment studied here features a standard amplification–propagation trade-off. A combination of the two ingredients is, therefore, necessary to provide a better fit to the data: implementation costs provide propagation and the taste shocks generate additional volatility. There is, however, a priori no reason to believe that this trade-off can be reconciled with the data in a way such that the dynamic oversynchronization between public consumption and the overall cycle is sufficiently, but not excessively, dampened.

We next study the role of implementation lags. The government decides now about G, not G'. The government flow budget constraint changes as[13]

$$\tau Y = G + \frac{\Omega}{2}(G - G_{-1})^2. \tag{9}$$

The third column of Table 5 displays the results of a model simulation where current G is decided on in the current period, but the parameters for implementation costs and the standard deviation of the taste shocks are fixed at their values from the baseline model with implementation lags. Without implementation lags the volatility of public consumption shoots up, its persistence goes down, and any correlation with private consumption at all horizons is dampened. Implementation lags thus play a similar role as implementation costs (see Table 4): they deliver propagation of public consumption. Implementation lags are, after all, an extreme form of implementation costs. Their main effect, however, is to get the rough shape of the correlogram between public consumption and private consumption/output right.

Since taking away implementation lags from the baseline model increases the volatility of public consumption and makes it less persistent, it can be expected that recalibration of Ω and ϵ_θ to minimize the same weighted quadratic form as the baseline model, but under the assumption of no implementation lags, will lead to a combination of higher implementation costs and/or lower variance of the taste shock. This is indeed the case: the recalibrated model (second column) has $\Omega = 45$ (up from $\Omega = 25$) and

[13] The symbol G_{-1} denotes last period's public consumption. Notice that for the computation, the public consumption that was decided on last period remains a state variable as long as $\Omega > 0$. Therefore, in the definition of the equilibrium functions, G replaces G' and G_{-1} replaces G as long as $\Omega > 0$. If $\Omega = 0$, then we have one state variable less.

$\epsilon_\theta = 0.005$ (down from $\epsilon_\theta = 0.006$). This model has volatility and persistence numbers for public consumption similar to the baseline model, and improves on the baseline model with respect to the correlogram between public consumption and output, but fails to deliver the dynamic correlogram between public consumption and private consumption. The intuition for this result can be seen by comparing the third and fourth columns in Table 5. In the baseline model, upon a positive productivity shock, output will increase contemporaneously, but public consumption cannot increase by construction. This explains the essentially zero contemporaneous correlation between output and public consumption. However, since private consumption and public consumption tend to move together because their marginal utilities are tied to the marginal utility of income, private agents will save today to ensure that the higher public consumption tomorrow is accompanied by higher private consumption tomorrow, hence the 0.409 contemporaneous correlation coefficient between public and private consumption. When implementation lags are removed, output and public consumption can better co-move, but public and private consumption are now also more reactive to the taste shocks that by themselves lead to opposite movements of both consumption types. If the reaction of government consumption is moved into the future by implementation lags, this effect is dampened because in expectation, taste shocks are mean-reverting. These differential effects on output correlation versus consumption correlation lead to an improvement of fit for the dynamic output correlogram, but a deterioration for the dynamic consumption correlogram, when implementation lags are removed.

However, the average deviation of the model-generated business cycle moments from their data counterparts as a fraction of their standard deviations is 1.56 in the recalibrated model with no implementation lags, whereas it is 1.40 in the baseline model, meaning that the baseline model with implementation lags has overall the better fit than the best fitting model without implementation lags. This is ultimately because with two types of implementation frictions, the model has a better chance of fitting the data: the implementation lags get the dynamic correlogram into roughly the right shape and the implementation costs deliver the persistence of government consumption. In a model without implementation lags, both aspects of public consumption dynamics have to be fitted using implementation costs only.[14]

4.3 *Alternative model specifications and robustness*

In this section, we discuss the sensitivity of our results to the specification of the felicity function over private consumption, public consumption, and leisure, and to the choice of the filtering method for extracting business cycles. In our baseline specification, the taste shock was directly between private and public consumption; see (1). There is another possible grouping of commodities in which the θ shock becomes a taste shock

[14]As Table 19 in Appendix D shows, the fact that the recalibrated model with no implementation lags can match the qualitative shape of the correlogram between public consumption and output is not robust to using a different filter to extract the cyclical component of the aggregate time series. With an HP smoothing parameter of 6.25 (see Ravn and Uhlig (2002)), the recalibrated model without implementation lags does worse in terms of fit to the data.

between public consumption and the private consumption *bundle* consisting of physical goods *as well as* leisure:

$$u(c, l, G) = \theta\big(\eta \log(c) + (1-\eta)\log(\tilde{l} - l)\big) + (1-\theta)\log(G). \tag{10}$$

In this specification, an increase in θ not only leads to a (persistent) expansion in private consumption, but also to a (persistent) reduction in labor supply and, therefore, output. This potentially means that a θ shock is a much more potent driver of aggregate fluctuations in this felicity specification than in the baseline specification.

This is indeed confirmed by comparing the third and the fifth columns of Table 6, which show that the model with the alternative felicity function under the same parameterization as the baseline model exhibits excess volatility of public consumption, 2.15%, as well as too low persistence, 0.492, both compared to the baseline model and the data. Thus, a lower ϵ_θ (from 0.006 to 0.004) and a higher Ω (from 25 to 35) in the recalibrated version of the alternative model is required (see the second column of Table 6). We also study a specification with perfectly inelastic labor supply that behaves very similarly to the baseline model. Inelastic labor supply only requires us to increase the volatility of the exogenous aggregate productivity shock necessary to match the volatility of output in the economy from 0.0123 in the baseline model to 0.0180 in the model with inelastic labor supply. As usual, elastic labor supply amplifies aggregate fluctuations.

Figure 1 sheds additional light on the role of endogenous labor supply in the felicity function for the propagation of the θ shocks into the economy. It shows for aggregate

Table 6. The role of the felicity function and labor supply.

Business Cycle Moment	Alternative Felicity Recalibrated	Alternative Felicity Param. from Baseline Model	Perfectly Inelastic Labor Supply	Baseline Model	Data
std(G)	1.329%	2.151%	1.079%	1.087%	1.796%
rho(G)	0.604	0.492	0.659	0.622	0.774
correl(G, Y)	0.044	0.054	0.090	0.105	0.237
correl(G, Y_{-1})	0.462	0.430	0.492	0.559	0.388
correl(G, Y_{-2})	0.465	0.356	0.520	0.546	0.379
correl(G, C)	0.340	0.247	0.376	0.409	0.534
correl(G, C_{-1})	0.395	0.174	0.358	0.449	0.582
correl(G, C_{-2})	0.325	0.148	0.312	0.364	0.484
std(C)	0.932%	0.932%	1.103%	0.979%	1.106%
rho(C)	0.592	0.568	0.519	0.543	0.629
correl(C, Y)	0.833	0.809	0.816	0.824	0.853
correl(C, Y_{-1})	0.641	0.617	0.591	0.605	0.510

Note: See the footnote to Table 2. The second column shows the results of a model where the felicity function over private consumption, public consumption, and leisure is given by (10) instead of (1). Implementation costs and the volatility of the relative taste shock between private and public consumption have been calibrated to minimize the same quadratic form as the baseline model: $\Omega = 35$, $\epsilon_\theta = 0.004$, $\bar{\theta} = 0.94$, and $\eta = 0.365$. The third column shows the results of a model where the felicity function over private consumption, public consumption, and leisure is given by (10) instead of (1), but the implementation costs parameter and the volatility of the relative taste shock are set equal to those from the baseline model: $\Omega = 25$, $\epsilon_\theta = 0.006$, and $\bar{\theta} = 0.94$. The fourth column shows the results of a model with inelastic labor supply, that is, $\eta = 1$. In this case, $\Omega = 50$, $\epsilon_\theta = 0.0008$, and $\bar{\theta} = 0.86$.

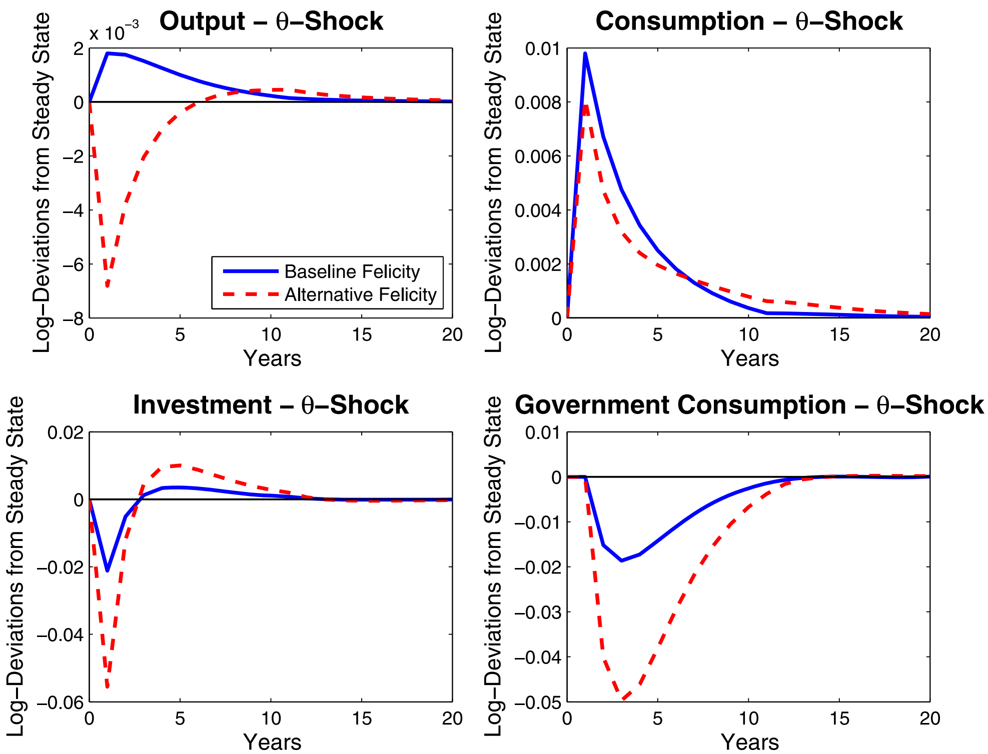

FIGURE 1. Theoretical impulse response functions to a θ shock. This figure shows the theoretical impulse responses—expressed in percentage deviations from a steady state—to the same θ shock for the baseline model (solid lines) and the model with the alternative felicity function (10) (dashed lines), both with $\Omega = 25$ and $\epsilon_\theta = 0.006$ (see Table 6 for details). Specifically, we set $z = 1$ and keep the economy at the lower value for θ until it reaches a steady state. We then increase θ to its upper value and let θ drop back probabilistically, according to its transition matrix. The reported IRF is the average over those time paths.

output, investment, and private and public consumption the theoretical impulse response functions (IRF's), in log deviations from the steady state, to a standardized taste shock toward private consumption (increase in θ) for the model with the baseline felicity and the model with the alternative felicity. In the baseline case, an increase in θ leads to an increase in labor supply and, therefore, leads contemporaneously to an increase in aggregate output. This means that public consumption does not need to fall as much to satisfy the increased taste in private consumption goods. Conversely, in the alternative specification, a positive taste shock toward the private consumption bundle leads to less labor supply and, therefore, contemporaneously leads to a fall in aggregate output, which is propagated through a reduction in capital accumulation. The effects of a taste switch on public consumption are more severe in this case and, therefore, the θ shock is more potent in generating fluctuations of government consumption.

Next, we discuss how the baseline model behaves in terms of aggregate fluctuations if we move away from the assumption of unit intratemporal substitutability in the felicity function. In other words, instead of using (1), we now explore a more general felicity function between private consumption, public consumption, and leisure:

$$u(c, l, G) = \frac{1}{1-\varrho} \log\bigl(\eta\bigl(\theta c^{1-\varrho} + (1-\theta)G^{1-\varrho}\bigr) + (1-\eta)(\tilde{l} - l)^{1-\varrho}\bigr). \tag{11}$$

Thus the intratemporal utility is of the constant elasticity of substitution (CES) type with the elasticity of substitution equal to $\frac{1}{\varrho}$. With this specification, it can be shown that c and G are Edgeworth substitutes (defined by a negative cross-derivative) if $\varrho < 1$, independent if $\varrho = 1$, and Edgeworth complements (defined by a positive cross-derivative) if $\varrho > 1$ (see Fiorito and Kollintzas (2004, Section 3.3), for a detailed discussion). The value $\varrho = 1$ constitutes our baseline calibration.

In Table 7 we show the statistics that characterize the aggregate dynamics of public and private consumption for both $\varrho = 1.5$ and $\varrho = 0.5$, with recalibrated Ω, ϵ_θ, and the standard deviation of the innovations to aggregate productivity.[15] Broadly speaking, the results are similar for the different ϱ specifications, especially in terms of the correlograms of public consumption with output and private consumption. The amplification–

Table 7. The role of intratemporal substitutability between c and G.

Business Cycle Moment	$\varrho = 1.5$	$\varrho = 0.5$	Baseline Model $\varrho = 1$	Data
std(G)	1.044%	1.297%	1.087%	1.796%
rho(G)	0.647	0.682	0.622	0.774
correl(G, Y)	0.067	0.159	0.105	0.237
correl(G, Y_{-1})	0.508	0.558	0.559	0.388
correl(G, Y_{-2})	0.529	0.564	0.546	0.379
correl(G, C)	0.423	0.364	0.409	0.534
correl(G, C_{-1})	0.416	0.390	0.449	0.582
correl(G, C_{-2})	0.345	0.347	0.364	0.484
std(C)	0.845%	1.254%	0.979%	1.106%
rho(C)	0.604	0.514	0.543	0.629
correl(C, Y)	0.774	0.852	0.824	0.853
correl(C, Y_{-1})	0.648	0.570	0.605	0.510

Note: See the footnote to Table 2. The second column shows the results of a model where the felicity function over private consumption, public consumption, and leisure is given by (11) with $\varrho = 1.5$ instead of (1). Implementation costs and the volatility of the relative taste shock between private and public consumption have been calibrated to minimize the same quadratic form as the baseline model: $\Omega = 40$, $\epsilon_\theta = 0.004$, $\bar{\theta} = 0.9353$, and $\eta = 0.3091$. The third column shows the results of a model where the felicity function over private consumption, public consumption, and leisure is given by (11) with $\varrho = 0.5$ instead of (1). Implementation costs and the volatility of the relative taste shock between private and public consumption have been calibrated to minimize the same quadratic form as the baseline model: $\Omega = 15$, $\epsilon_\theta = 0.008$, $\bar{\theta} = 0.6836$, and $\eta = 0.5373$.

[15] We choose to conduct a scenario analysis that allows for substitutability, independence, and complementarity, because the empirical evidence for ϱ is mixed and often depends on the particular data set and statistical methods used. For example, Campbell and Mankiw (1990) pointed toward an independence relationship in U.S. data, whereas Evans and Karras (1998) showed evidence that nonmilitary spending and private consumption in a data set of 66 countries are substitutes. Fiorito and Kollintzas (2004), on the other hand, found complementarity between aggregate government consumption and aggregate private consumption in European countries.

propagation trade-off is somewhat more easily resolved when public and private consumption are Edgeworth substitutes ($\varrho = 0.5$).

The intuition for this result is that both private and public consumption react more to an aggregate productivity shock (as can be seen in Table 7), because they, as a bundle, are now substitutes relative to leisure, which means they are more sensitive to a given increase in the real wage.[16] This is can also be seen in Table 9, which shows that with $\varrho = 0.5$, the contribution of aggregate productivity shocks to fluctuations in public consumption is largest in all the models we study.

In Table 8, we discuss how our conclusions about the aggregate dynamics of public and private consumption depend on the filter used to extract the cyclical component in the data. While an HP smoothing parameter of 100 is commonly used for annual data, Ravn and Uhlig (2002) advocated a smaller value, 6.25, leading to a more flexible trend component. The basic result in terms of overall quality of fit does not change, except that with an HP(6.25) filter the baseline calibration does much better in matching the volatility of public consumption in the data.[17]

Table 8. The role of filtering.

Business Cycle Moment	HP(6.25)		HP(100)	
	Model	Data	Baseline Model	Data
std(G)	0.699%	0.783%	1.087%	1.796%
rho(G)	0.230	0.296	0.622	0.774
correl(G, Y)	−0.136	0.003	0.105	0.237
correl(G, Y_{-1})	0.564	0.306	0.559	0.388
correl(G, Y_{-2})	0.307	0.375	0.546	0.379
correl(G, C)	0.184	0.217	0.409	0.534
correl(G, C_{-1})	0.300	0.302	0.449	0.582
correl(G, C_{-2})	0.121	0.320	0.364	0.484
std(C)	0.569%	0.705%	0.979%	1.106%
rho(C)	0.124	0.362	0.543	0.629
correl(C, Y)	0.838	0.862	0.824	0.853
correl(C, Y_{-1})	0.258	0.223	0.605	0.510

Note: See the footnote to Table 2. Recalibrated parameters for the HP(6.25) case are $\Omega = 15$ and $\epsilon_\theta = 0.005$. Otherwise the HP(6.25) case is identical to the baseline.

[16] Of course, this also means that for the same volatility of aggregate productivity, households are more willing to forgo leisure upon positive productivity shocks and output would become more volatile, which is why we have to recalibrate the conditional volatility of aggregate productivity from 0.0123 to 0.0095 for the model to continue to match the output volatility in the data. Indeed, then the volatility of investment relative to the baseline model declines from 6.370% to 5.720%. Similarly, when private and public consumption are substitutes, so are private consumption and leisure, which means that for a given taste shock toward private consumption, the output response is larger, the smaller is ϱ, and thus the negative effect on public consumption is mitigated, relative to the baseline case with $\varrho = 1$ shown in Figure 1. Thus the potency of the taste shock to generate fluctuations in public consumption is reduced, which, in turn, means that in the recalibrated model, the volatility of the taste shock has to be increased from $\epsilon_\theta = 0.006$ to $\epsilon_\theta = 0.008$.

[17] The Simple Model without taste shocks, implementation lags, or costs under the HP(6.25) filter also matches the volatility of public consumption in the data almost perfectly, but delivers zero persistence and again the wrong dynamic correlogram with output/private consumption for public consumption.

Table 9. Variance decomposition: Various models.

Specification	Contribution of z Shocks	Contribution of θ Shocks	Both
Baseline	49.58%	40.80%	90.38%
No implementation lag	34.90%	56.02%	90.92%
Alternative felicity	28.15%	68.45%	96.60%
Inelastic labor supply	46.81%	53.91%	100.72%
$\varrho = 1.5$	46.52%	48.60%	95.12%
$\varrho = 0.5$	65.39%	29.61%	95.01%
HP(6.25)	40.82%	49.49%	90.31%

Note: See the footnotes to Tables 2, 3, 6, 7, and 8.

A possible interpretation is that public consumption data have lower frequency movements that are filtered out by an HP(6.25) filter, but not by an HP(100) filter. Our model is neither able nor designed to capture these medium-term fluctuations of public consumption in the data.[18]

Finally, Table 9 shows, for the various alternative specifications discussed in this section, the decomposition of the variance of public consumption in the corresponding model with both aggregate productivity and taste shocks between private and public consumption, and variants for each model where one of these shocks is shut down.

5. Conclusion

We document the business cycle behavior of various subaggregates of government purchases, in particular, state and local government consumption. We provide a tractable workhorse model that is as close as possible to standard quantitative macroeconomic models so as to generate a good fit to the business cycle features of public consumption. We argue that both implementation lags and implementation costs in the budgeting process plus taste shocks for public consumption relative to private consumption are essential to generate this fit. We then use this model to decompose the variance of public consumption into fluctuations that are endogenous responses of the policy maker to changing macroeconomic conditions versus fluctuations that are the direct result of taste shocks in the populace between private and public consumption. In our baseline specification and using the best fitting model within this model class, 50% of the variance of public consumption is explained by aggregate productivity shocks. Some model features used here are rather stylized and need a better microfoundation, which we leave for future research.

[18] Tables 18 and 19 in Appendix D are the analogues of Tables 4 and 5 in Section 4.2 and show, respectively, the role of the preference shocks, implementation costs, and implementation lags for the fit of the model under the alternative filtering assumption. The bottom line is that the analysis in Section 4.2 holds and, if anything, the necessity of preference shocks, implementation costs, and implementation lags for the fit of the model is starker.

Appendix A: Data Appendix

Table 10. Business cycle facts: government purchases with functional disaggregation.

Moment	rho(·) First Order	correl(·, Y)	correl(·, Y_{-1})	correl(·, Y_{-2})	Frac. of GSL
General public service	0.681	0.256	0.280	0.197	10.72%
Public order and safety	0.513	0.084	0.403	0.544	13.90%
Economic affairs	0.697	0.535	0.582	0.348	19.40%
Transportation	0.676	0.565	0.545	0.310	15.17%
Other economic affairs	0.538	0.317	0.527	0.354	4.20%
Housing & comm. serv.	0.410	0.259	0.496	0.644	3.77%
Health	0.700	−0.302	−0.015	0.217	3.51%
Recreation and culture	0.552	0.049	0.454	0.542	1.98%
Education	0.798	0.502	0.576	0.417	42.98%
Elementary and secondary	0.782	0.488	0.535	0.414	34.90%
Higher	0.626	0.460	0.541	0.280	6.57%
Libraries and other	0.646	0.352	0.554	0.496	1.76%
Income security	0.703	0.093	0.155	0.171	3.88%

Note: The data source is the BEA (NIPA data). All variables are annual and the sample goes from 1960 to 2006. They are deflated by their corresponding deflators, and logged and filtered with a Hodrick–Prescott filter with smoothing parameter 100. The term rho(·) denotes the first-order autocorrelation of an aggregate variable. correl(·, Y) denotes the contemporaneous correlation with aggregate GDP, and correl(·, Y_{-1}) and correl(·, Y_{-2}) denote the correlation with aggregate 1- and 2-year lagged GDP, respectively. Frac. of GSL denotes the fraction of the corresponding aggregate with respect to total state and local government purchases (there is no consumption/investment distinction in the functional disaggregation). Housing & comm. serv. stands for housing and community services.

Table 11. Business cycle facts: government consumption with HP(6.25).

Moment	GSLC	GNDC	GND	GC	G	CNDS
std(·)	0.783%	0.777%	0.943%	1.173%	1.362%	0.705%
rho(·)	0.296	0.218	0.412	0.461	0.534	0.362
correl(·, Y)	0.003	−0.013	0.235	−0.034	0.049	0.862
correl(·, Y_{-1})	0.306	0.209	0.428	0.206	0.340	0.223
correl(·, Y_{-2})	0.375	0.364	0.397	0.390	0.433	−0.283
correl(·, CNDS)	0.217	0.104	0.263	−0.056	0.016	—
correl(·, CNDS_{-1})	0.302	0.199	0.430	0.150	0.292	—
correl(·, CNDS_{-2})	0.320	0.381	0.438	0.446	0.480	—

Note: The data source is the BEA (NIPA data). All variables are annual and the sample goes from 1960 to 2006. They are deflated by their corresponding deflators, and logged and filtered with a Hodrick–Prescott filter with smoothing parameter 6.25. GSLC stands for state and local government consumption. GNDC denotes total nondefense consumption, GND denotes total nondefense purchases, and GC denotes total government consumption. The term G denotes total government purchases. The term std(·) denotes the time-series volatility of an aggregate variable, rho(·) denotes its first-order autocorrelation, correl(·, Y) denotes the contemporaneous correlation with aggregate GDP, and correl(·, Y_{-1}) and correl(·, Y_{-2}) denote the correlation with aggregate 1- and 2-year lagged GDP, respectively. The term CNDS stands for nondurable and services consumption.

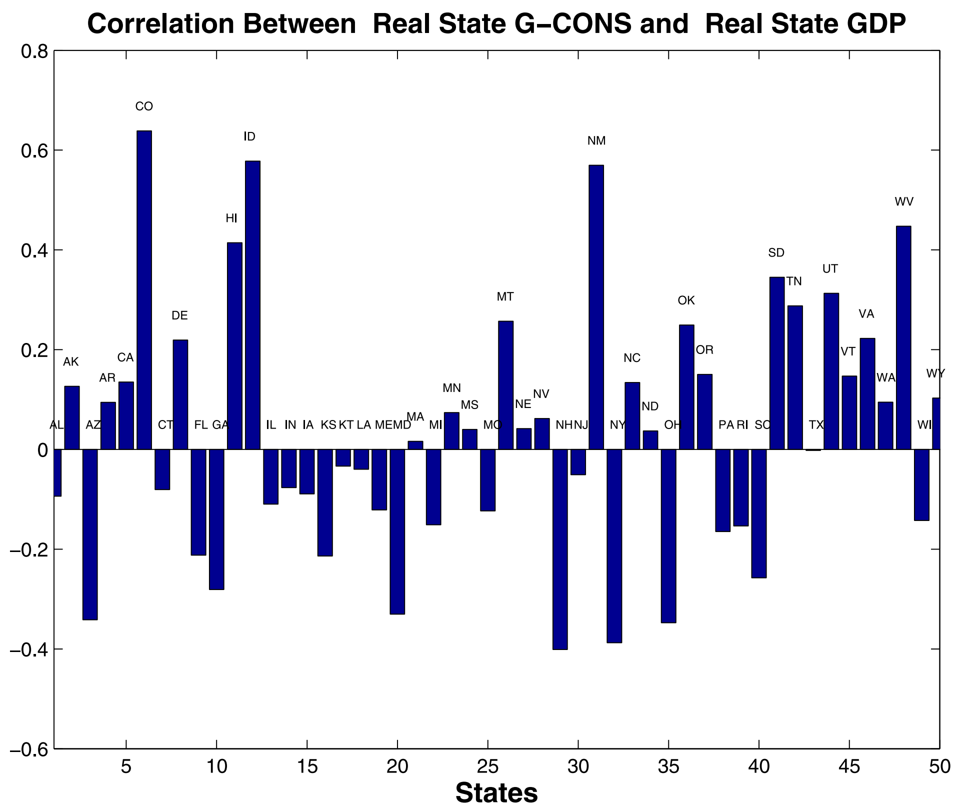

Figure 2. Contemporaneous correlation of GDP and public consumption by state. Real GDP by state is taken from the BEA. Public consumption by state is measured as the "Total Current Operations" category from the Annual Survey of State Government Finances from the Census, which we deflate by a state-specific deflator for government purchases, computed from BEA data on total nominal and real government purchases. All variables are annual and the sample goes from 1977 to 2006. They are logged and filtered with a Hodrick–Prescott filter with smoothing parameter 100.

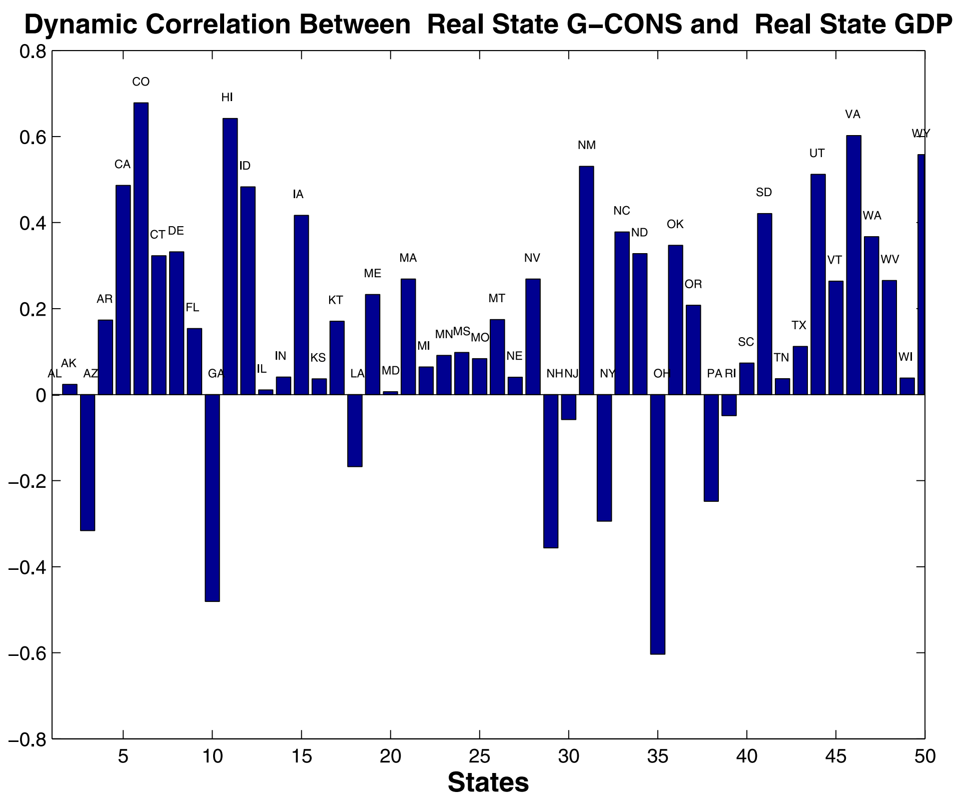

FIGURE 3. Dynamic correlation of GDP (1 year lagged) and public consumption by State. See the caption to Figure 2.

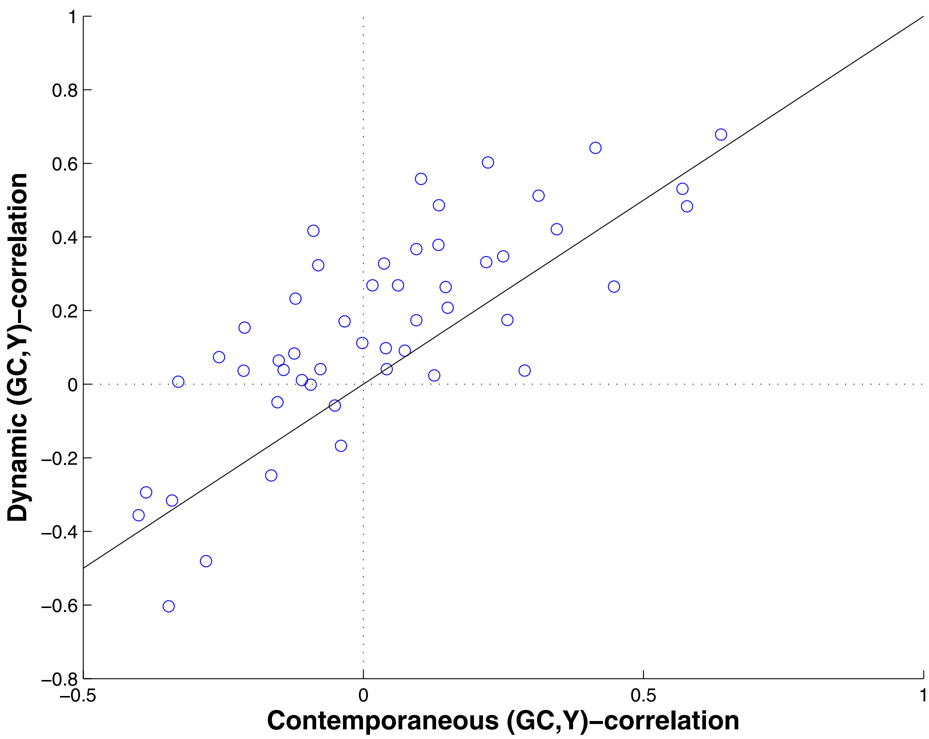

FIGURE 4. Dynamic correlation of GDP (1 year lagged) and public consumption versus the contemporaneous correlation. See the caption to Figure 2. Each point represents a U.S. state and the line is a 45-degree line.

Appendix B: Numerical Appendix

Computational algorithm

We solve the Markov perfect equilibrium using a fixed point iteration procedure from (H, Ψ) onto itself. The algorithm can be summarized as follows (for the baseline case).

Algorithm 1 (Fixed Point Iteration on (H, Ψ)).
Step 0: Fix the functional forms for H and Ψ. Start from an initial guess of coefficients $\{a_0^0, \dots, a_6^0\}$ and $\{b_0^0, \dots, b_2^0\}$ to get the initially conjectured functions (H^0, Ψ^0). Set up a convergence criterion ε for equilibrium iteration on the coefficients $(\mathbf{a}, \mathbf{b})$. Select the interpolation grid for (k, K, G) used in the spline approximation of both household's continuation value function (v) and its best response value function (J). In addition, specify the interpolation grid for G', $(G_i')_{i=1}^{N_G}$, used for the spline approximation of J.
Step 1: In the equilibrium iteration loop $n \geq 0$, imposing (H^n, Ψ^n) in the household's optimization problem, use value function iteration to solve the household's parametric dynamic programming problem. For each (z, θ), use interpolation on the (k, K, G) dimensions to get the spline approximation for the continuation value function as $v^n(k, K, G, z, \theta; \Psi^n, H^n)$.
Step 2: Without imposing Ψ^n and instead fixing G' on the prespecified N_G grid points of G', $(G_i')_{i=1}^{N_G}$, use H^n and v^n to solve for the best-response value (J) and decision (h) for each combination of grid points of (k, K, G, G') and shock (z, θ). For each (z, θ), use interpolation on the (k, K, G, G') dimensions to get the spline approximation for J and h as $J^n(k, K, G, z, \theta, G'; \Psi^n, H^n)$ and $h^n(k, K, G, z, \theta, G'; \Psi^n, H^n)$.
Step 3: Simulate the economy using $N_H = 1$ households and T periods. In each period t of the simulation, calculate the equilibrium policy $G_t^{\prime\text{eq.}}$ by maximizing the spline $J^n(K, K, G, z, \theta, G'; \Psi^n, H^n)$ over the G' dimension. Calculate the best-response decision based on $h^n(K, K, G, z, \theta, G'; \Psi^n, H^n)$ for both equilibrium $G_t^{\prime\text{eq.}}$ and the N_G grid points $(G_i')_{i=1}^{N_G}$. Gather a time series of data points $(K_{t+1}^{\text{eq.}}, (K_{t+1,i})_{i=1}^{N_G}, G_t^{\prime\text{eq.}}, (G_{t,i}' = G_i')_{i=1}^{N_G})_{t=1}^{T}$, that is, capital statistics both on $(K_{t+1}^{\text{eq.}})$ and off the equilibrium path $((K_{t+1,i})_{i=1}^{N_G})$, with a total sample size of $T(1 + N_G)$.
Step 4: Use the gathered time series to get—separately for each value of the (z, θ) grid—ordinary least squares (OLS) estimates of $\{\widehat{a}_0^n, \dots, \widehat{a}_6^n\}$, $\{\widehat{b}_0^n, \dots, \widehat{b}_2^n\}$, which, with a slight abuse of notation, we summarize as $(\widehat{H}^n, \widehat{\Psi}^n)$. Notice that $\widehat{H}^n$ is updated on both the on- and off-equilibrium paths; $\widehat{\Psi}^n$ is updated only on the equilibrium path.
Step 5: If $|H^n - \widehat{H}^n| < \varepsilon$ and $|\Psi^n - \widehat{\Psi}^n| < \varepsilon$, go to Step 6. Otherwise, set

$$H^{n+1} = \alpha_H \times \widehat{H}^n + (1 - \alpha_H) \times H^n,$$
$$\Psi^{n+1} = \alpha_\Psi \times \widehat{\Psi}^n + (1 - \alpha_\Psi) \times \Psi^n,$$

with $\alpha_H, \alpha_\Psi \in (0, 1]$, and go back to Step 1.[19]

[19]We choose $\varepsilon = 10^{-4}$ and $T = 10{,}000$, of which we discard the first 500 observations, when we update the transition and policy rules or compute summary statistics. To eliminate sampling error, we use the same series of aggregate shocks for all iterations in the algorithm and across all model simulations.

TABLE 12. The equilibrium law of motion for capital: baseline model, equation (7).

Parameter	$z = 0.9384$	$z = 0.9687$	$z = 1$	$z = 1.0323$	$z = 1.0657$
			$\theta^l = 0.8460$		
$a_0(\cdot, \theta_1)$	−1.0907	−0.9731	−0.8780	−0.7879	−0.7209
$a_1(\cdot, \theta_1)$	0.8544	0.8549	0.8536	0.8514	0.8472
$a_2(\cdot, \theta_1)$	0.1785	0.1705	0.1640	0.1568	0.1492
$a_3(\cdot, \theta_1)$	−1.2501	−1.1409	−1.0555	−0.9741	−0.9160
$a_4(\cdot, \theta_1)$	−0.4476	−0.4127	−0.3856	−0.3600	−0.3410
$a_5(\cdot, \theta_1)$	−0.0438	−0.0404	−0.0377	−0.0352	−0.0335
$a_6(\cdot, \theta_1)$	0.0805	0.0782	0.0764	0.0746	0.0721
			$\theta^h = 0.8563$		
$a_0(\cdot, \theta_2)$	−1.0861	−0.9918	−0.8930	−0.7977	−0.7194
$a_1(\cdot, \theta_2)$	0.8558	0.8557	0.8538	0.8517	0.8478
$a_2(\cdot, \theta_2)$	0.1772	0.1691	0.1633	0.1559	0.1476
$a_3(\cdot, \theta_2)$	−1.2421	−1.1540	−1.0665	−0.9799	−0.9102
$a_4(\cdot, \theta_2)$	−0.4447	−0.4162	−0.3884	−0.3611	−0.3385
$a_5(\cdot, \theta_2)$	−0.0435	−0.0407	−0.0380	−0.0353	−0.0332
$a_6(\cdot, \theta_2)$	0.0799	0.0777	0.0760	0.0740	0.0714

Note: This table displays the coefficients for the equilibrium law of motion for the (natural logarithm of the) aggregate capital stock, equation (7), for the baseline case. Recall equation (7):

$$\log K' = a_0(z, \theta) + a_1(z, \theta)\log K + a_2(z, \theta)\log G + a_3(z, \theta)\log G' + a_4(z, \theta)(\log G')^2 + a_5(z, \theta)(\log G')^3 + a_6(z, \theta)\log G \log G'.$$

TABLE 13. The equilibrium government policy function for G': baseline model, equation (8).

Parameter	$z = 0.9384$	$z = 0.9687$	$z = 1$	$z = 1.0323$	$z = 1.0657$
			$\theta^l = 0.8460$		
$b_0(\cdot, \theta_1)$	−1.6378	−1.6080	−1.5947	−1.6041	−1.6168
$b_1(\cdot, \theta_1)$	0.1806	0.1880	0.2061	0.2280	0.2399
$b_2(\cdot, \theta_1)$	0.4546	0.4614	0.4626	0.4559	0.4470
			$\theta^h = 0.8563$		
$b_0(\cdot, \theta_2)$	−1.6258	−1.6076	−1.5896	−1.5929	−1.6019
$b_1(\cdot, \theta_2)$	0.1796	0.1863	0.2032	0.2272	0.2406
$b_2(\cdot, \theta_2)$	0.4638	0.4667	0.4694	0.4649	0.4575

Note: This table displays the coefficients for the equilibrium government policy function for (the natural logarithm of) tomorrow's government consumption, equation (8), for the baseline case. Recall equation (8): $\log G' = b_0(z, \theta) + b_1(z, \theta)\log K + b_2(z, \theta)\log G$.

Table 14. Different laws of motion.

Business Cycle Moment	Baseline Model	Linear Law of Motion	Data
std(G)	1.087%	1.207%	1.796%
rho(G)	0.622	0.602	0.774
correl(G, Y)	0.105	0.147	0.237
correl(G, Y_{-1})	0.559	0.625	0.388
correl(G, Y_{-2})	0.546	0.555	0.373
correl(G, C)	0.409	0.420	0.534
correl(G, C_{-1})	0.449	0.484	0.582
correl(G, C_{-2})	0.364	0.361	0.484
std(C)	0.979%	0.977%	1.106%
rho(C)	0.543	0.527	0.629
correl(C, Y)	0.824	0.839	0.853
correl(C, Y_{-1})	0.605	0.589	0.510

Note: See the footnote to Table 2. The second column displays the results for the same parameters as the Baseline Model, except that the equilibrium law of motion for the (natural logarithm of the) aggregate capital stock, equation (7), only contains the first four (i.e., linear) terms with coefficients a_0 to a_3.

Table 15. Weighting.

Business Cycle Moment	Weighting	Data
std(G)	0.318	1.796%
rho(G)	0.077	0.774
correl(G, Y)	0.229	0.237
correl(G, Y_{-1})	0.214	0.388
correl(G, Y_{-2})	0.173	0.379
correl(G, C)	0.167	0.534
correl(G, C_{-1})	0.182	0.582
correl(G, C_{-2})	0.136	0.484
std(C)	0.128	1.106%
rho(C)	0.096	0.629
correl(C, Y)	0.033	0.853
correl(C, Y_{-1})	0.033	0.510

Note: See the footnote to Table 2. The second column displays the standard deviations of the 12 business cycle moments used for the matching exercise from 2,000 nonparametric bootstrap simulations for GDP, private consumption (CNDS), and public consumption (GSLC) with nonoverlapping blocks of 8 years. They are the weighting coefficients in the quadratic form to be minimized (see Section 3.3).

Step 6: Check the $R2$ of the final OLS regressions. If the $R2$ is high enough to convey confidence that the true equilibrium rule is well approximated, stop. Otherwise, go back to Step 0 and choose more flexible functional forms for H and Ψ.

Now we describe the algorithm in more detail. In Step 1, we iterate on the value function until it converges at a set of collocation points, which are chosen to be the grid points of (k, K, G) defined in Step 0. In each step of the value function iteration, we use a multi-dimensional cubic spline with the aforementioned interpolation grid to approximate the continuation value function. For each collocation point of current state variables (k, K, G) and exogenous aggregate state variables (z, θ), we use (H^n, Ψ^n) to infer the values of K' and G', which, in turn, allows us to compute numerically aggregate labor usage L (given the aggregate resource constraint and the intratemporal first-order condition of the household).

Given the knowledge of aggregate variables (K, K', G, G', L), the Bellman equation can be maximized numerically along the k' dimension through a golden section search method. We can solve analytically for the individual labor–leisure choice, (l), using the intratemporal first-order condition (for each possible k'). In our numerical implementation, we find that the golden section search method sometimes proves to be more robust than derivative-based methods and provides accurate solutions. The same golden section search method is used in the numerical optimization part in Steps 2 and 3, where we use interpolation steps to compute—now allowing for a continuous choice—the optimal G'.

Appendix C: Commitment Appendix

In the Ramsey Model, the sequential competitive equilibrium for a given choice of a feasible policy path, $\{G_t\}_{t=0}^{\infty}$, is characterized by

$$
\begin{aligned}
&\frac{\partial u(C_t, L_t, G_t)}{\partial C} = \beta E\left[\frac{\partial u(C_{t+1}, L_{t+1}, G_{t+1})}{\partial C}\big(1-\delta+(1-\tau_{t+1})r_{t+1}\big)\right],\\
&-\frac{\partial u(C_t, L_t, G_t)}{\partial L} = \frac{\partial u(C_t, L_t, G_t)}{\partial C}(1-\tau_t)w_t,\\
&C_t + G_t + K_{t+1} = (1-\delta)K_t + z_t F(K_t, L_t),\\
&\tau_t = \frac{G_t}{z_t F(K_t, L_t)},
\end{aligned}
$$

where the first two lines are the household's Euler equation and intratemporal first-order condition, respectively, the third equation is the resource constraint, and the last line is the government budget constraint. Using quantity variables to substitute out price variables, that is,

$$
\begin{aligned}
&(1-\tau_t)r_t = \left(1-\frac{G_t}{Y_t}\right)\left(\alpha\frac{Y_t}{K_t}\right) = \alpha\frac{Y_t - G_t}{K_t},\\
&(1-\tau_t)w_t = \left(1-\frac{G_t}{Y_t}\right)\left((1-\alpha)\frac{Y_t}{L_t}\right) = (1-\alpha)\frac{Y_t - G_t}{L_t},
\end{aligned}
$$

we get the primal approach to a social planner's problem as

$$
\begin{aligned}
&\max_{\{C_t, L_t, G_t, K_{t+1}\}_{t=0}^{\infty}} E_0\left[\sum_{t=0}^{\infty}\beta^t u(C_t, L_t, G_t)\right] \\
&\text{s.t.}\quad \frac{\partial u(C_t, L_t, G_t)}{\partial C} \\
&\qquad = \beta E\left[\frac{\partial u(C_{t+1}, L_{t+1}, G_{t+1})}{\partial C}\left(1-\delta+\alpha\frac{Y_{t+1}-G_{t+1}}{K_{t+1}}\right)\right], \\
&\qquad \frac{\partial u(C_t, L_t, G_t)}{\partial L} = \frac{\partial u(C_t, L_t, G_t)}{\partial C}(1-\alpha)\frac{Y_t - G_t}{L_t}, \\
&\qquad C_t + G_t + K_{t+1} = (1-\delta)K_t + z_t F(K_t, L_t).
\end{aligned}
$$

Following Marcet and Marimon (2011), the sequential problem can be written in the form of a recursive Lagrangian, which leads to the saddle-point functional equation (SPFE)

$$
\begin{aligned}
v(K, \mu, z) = \min_{\mu'} \max_{\{C, L, G, K'\}} &\Bigg\{u(C, L, G) \\
&+ \frac{\partial u(C, L, G)}{\partial C}\left[\mu\left(1-\delta+\alpha\frac{zF(K, L)-G}{K}\right)-\mu'\right] \\
&+ \beta E\big[v\big(K', \mu', z'\big)|z\big]\Bigg\} \\
\text{s.t.}\quad &\frac{\partial u(C, L, G)}{\partial L} = \frac{\partial u(C, L, G)}{\partial C}(1-\alpha)\frac{zF(K, L)-G}{L}, \\
&C + G + K' = (1-\delta)K + zF(K, L),
\end{aligned}
$$

where $\mu \in \mathbb{R}$ and $\mu' \in \mathbb{R}$ are the Lagrange multipliers for the household Euler equation from the previous and current period, respectively. The decision rule is a time-invariant function $(K', \mu', L, C, G) = \Xi(K, \mu, z)$. The time-series path for the solution can be obtained by iterating on the decision rule, with the initial state variables set at $K = K_0$ and $\mu_0 = 0$.

We again use a value function iteration approach to solve the SPFE. The numerical implementation closely follows the procedure described in Appendix B, with the main difference being that the optimization stage now involves both a minimization and a maximization step. In particular, in each step of the iteration we use a grid search method along the μ' dimension to solve the minimization step. For any given μ' grid point, the maximization step is then solved through a sequential quadratic programming method. We found that the grid search method is the most robust method across different specifications, especially when a good initial guess for the minimizing μ' is not available. In addition, the grid search method can be easily and efficiently implemented using parallel computation.

Appendix D: Results Appendix

Table 16. Baseline result: statistics for $\frac{G}{Y}$ and $\frac{C}{Y}$ ratios.

Business Cycle Moment	Baseline Model	Data
std($\frac{G}{Y}$)	0.283	0.608
rho($\frac{G}{Y}$)	0.544	0.900
correl($\frac{G}{Y}$, Y)	−0.761	−0.244
correl($\frac{G}{Y}$, Y_{-1})	−0.110	−0.027
correl($\frac{G}{Y}$, Y_{-2})	0.192	0.192
correl($\frac{G}{Y}$, C)	−0.486	−0.058
correl($\frac{G}{Y}$, C_{-1})	0.029	0.094
correl($\frac{G}{Y}$, C_{-2})	0.303	0.283
std($\frac{C}{Y}$)	1.086	0.967
rho($\frac{C}{Y}$)	0.558	0.701
correl($\frac{C}{Y}$, Y)	−0.803	−0.702
correl($\frac{C}{Y}$, Y_{-1})	−0.185	−0.370

Note: See the footnotes to Tables 1 and 2. Ratios are filtered with a linear trend instead of HP(100).

Table 17. Baseline result: other second moments.

Business Cycle Moment	Baseline Model	Data
std(Y)	1.936%	1.897%
rho(Y)	0.400	0.541
std(I)	6.370%	7.843%
rho(I)	0.317	0.420
correl(I, Y)	0.949	0.84
std(L)	0.968%	1.776%
rho(L)	0.318	0.610
correl(L, Y)	0.925	0.814

Note: See the footnotes to Tables 1 and 2. The variable I is real private gross fixed investment from the NIPA data; L is total nonfarm payroll employment from the Bureau of Labor Statistics (BLS) monthly data averaged to the annual frequency.

Table 18. The role of implementation costs and taste shocks: HP(6.25).

Business Cycle Moment	Baseline Model	No Taste Shock	No Implementation Costs	No Taste Shock, No Implementation Costs	No Productivity Shock
std(G)	0.699%	0.446%	1.221%	0.707%	0.464%
rho(G)	0.230	0.322	−0.013	0.085	0.218
correl(G, Y)	−0.136	−0.204	−0.059	−0.052	0.057
correl(G, Y_{-1})	0.564	0.841	0.568	0.974	−0.946
correl(G, Y_{-2})	0.307	0.461	0.096	0.149	−0.302
correl(G, C)	0.184	0.154	0.150	0.248	0.301
correl(G, C_{-1})	0.300	0.964	0.163	0.965	−0.848
correl(G, C_{-2})	0.121	0.361	0.025	−0.003	−0.367
std(C)	0.569%	0.511%	0.564%	0.505%	0.309%
rho(C)	0.124	0.180	0.115	0.165	−0.093
correl(C, Y)	0.838	0.931	0.839	0.935	−0.207
correl(C, Y_{-1})	0.258	0.295	0.239	0.277	−0.345

Note: See the footnote to Table 2. The Baseline Model features both a 1-year implementation lag and implementation costs ($\Omega = 15$), $\epsilon_\theta = 0.005$. The No Taste Shock model is identical to the Baseline Model, but sets $\epsilon_\theta = 0$. The No Implementation Costs model is identical to the Baseline Model, but sets $\Omega = 0$. The No Taste Shock–No Implementation Costs model is a combination of the third and fourth columns. The No Productivity Shock model is the same as the Baseline Model, but without aggregate productivity shocks.

Table 19. The role of implementation lags: HP(6.25).

Business Cycle Moment	No Implementation Lag Recalibrated	No Implementation Lag Param. from Baseline	Baseline Model	Data
std(G)	0.718%	1.033%	0.699%	0.783%
rho(G)	0.257	0.193	0.230	0.296
correl(G, Y)	0.387	0.322	−0.136	0.003
correl(G, Y_{-1})	0.280	0.202	0.564	0.306
correl(G, Y_{-2})	0.070	0.030	0.307	0.375
correl(G, C)	0.178	−0.031	0.184	0.217
correl(G, C_{-1})	0.122	0.034	0.300	0.302
correl(G, C_{-2})	0.019	0.036	0.121	0.320
std(C)	0.537%	0.562%	0.569%	0.705%
rho(C)	0.142	0.116	0.124	0.362
correl(C, Y)	0.851	0.823	0.838	0.862
correl(C, Y_{-1})	0.277	0.253	0.258	0.223

Note: See the footnote to Table 2. The second column shows the results of a model where public consumption is decided on contemporaneously, but implementation costs and the volatility of the relative taste shock between private and public consumption have been calibrated to minimize the same quadratic form as the Baseline Model: $\Omega = 25$, $\epsilon_\theta = 0.004$. The third column shows the results of a model where public consumption is decided on contemporaneously, but the implementation costs parameter and the volatility of the relative taste shock are set equal to those in the Baseline Model: $\Omega = 15$, $\epsilon_\theta = 0.005$.

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