Quantitative Economics 3 (2012), 177–209 1759-7331/20120177

Labor income profiles are not heterogeneous: Evidence from income growth rates

Dmytro Hryshko
Department of Economics, University of Alberta

Idiosyncratic labor incomes are typically modeled either by stochastic processes with heterogeneous income profiles (HIPs) or restricted income profiles (RIPs). The HIP assumes that individual labor income grows deterministically at an unobserved rate and contains a persistent but stationary component, while the RIP assumes that income contains a random walk, a stationary component, and no unobserved deterministic growth-rate component. I show that if idiosyncratic labor income contains a persistent component, a deterministic household-specific trend, and a random-walk component, then all of the components can be identified in small unbalanced panels. Using data on idiosyncratic labor income growth from the Panel Study of Income Dynamics, I find that the estimated variance of deterministic income growth is zero, that is, the HIP model can be rejected. The RIP model with a permanent component cannot be rejected. This result is important for an appropriate choice of modeling the heterogeneity in individual incomes and calibrating/estimating macromodels with incomplete insurance markets and heterogeneous agents.

Keywords. Idiosyncratic income processes, heterogeneity, labor income risk.
JEL classification. J31, D91, E21.

1. Introduction

Individuals and households face substantial amounts of idiosyncratic labor market risk. Layoffs, health shocks, bonuses, demotions, and time-varying returns to the individual skills valued by the labor market contribute toward fluctuating individual labor incomes. If credit and insurance markets are not functioning perfectly, idiosyncratic labor income risk will affect individual and aggregate welfare.

Two different approaches to modeling individual and household labor income risks currently stand out. The first approach, which has a long-standing tradition, models

Dmytro Hryshko: dhryshko@ualberta.ca
I benefited from conversations with and/or comments from Adolf Buse, Flavio Cunha, Fatih Guvenen, Jonathan Halket, Gueorgui Kambourov, Greg Kaplan, María Luengo-Prado, Iourii Manovskii, René Morissette, Giuseppe Moscarini, Bent Sørensen, Henry van Egteren, and Gianluca Violante. For useful comments, I am also grateful to the editor, three anonymous referees, conference participants at the 2008 SED Meetings in Boston, the 2008 CEA Meetings in Vancouver, and the 2009 NBER Summer Institute (Consumption group), and seminar participants at the EERC in Kiev, the University of Pennsylvania, the University of British Columbia, UCL, and the Federal Reserve Bank of St. Louis. This paper was previously circulated under the title "RIP to HIP: The data reject heterogeneous labor income profiles."

DOI: 10.3982/QE112

each individual's income as growing at the individual-specific, deterministic rate, with the level of income affected by a stochastic component with moderate persistence. Since each individual's labor income profile, even in the absence of shocks, is unique, I label this model, following Guvenen (2009), the heterogeneous income profiles (HIP) model. The second approach models idiosyncratic labor income as the sum of a permanent random-walk component, the shocks to which persist for the entire working lifetime of an individual, and a mean-reverting stationary component, the shocks to which die out quickly. Since this model abstracts from the deterministic growth-rate heterogeneity, I label it the restricted income profiles (RIP) model. Even though variants of the RIP are currently a preferred choice in macromodels, there is no consensus in the labor income processes literature on which income model best fits the earnings data. As Guvenen (2007) concluded, "…it is fair to say that this literature has not produced an unequivocal verdict." This paper is a step toward finding a verdict in favor of the RIP model.

I start with a general income model that encompasses the RIP and the HIP models. I then conduct a Monte Carlo study on small unbalanced panels to explore identification of different income processes found in the literature, obtained when certain restrictions on this general process are imposed. Importantly, the samples utilized in the Monte Carlo analysis replicate my empirical sample from the Panel Study of Income Dynamics (PSID), the data set typically used in the literature, in terms of the number of individuals, the number of person-year observations, and the cross-sectional distribution of age by year. I find that if the true income process is the RIP with a permanent random-walk component and an econometrician estimates the misspecified HIP model instead, he will typically find statistically significant amounts of growth-rate heterogeneity, of magnitudes comparable with those in the literature. The results of a Monte Carlo study confirm that the parameters of the general income process, composed of a deterministic growth rate, a permanent random walk, and transitory components, should be precisely recovered using the autocovariance moments of income growth rates. This contrasts with the concern raised recently (see, e.g., Guvenen (2009)) that the autocovariance moments of income growth rates are not informative enough for identifying the growth-rate heterogeneity in small samples.[1]

I then proceed by estimating the model utilizing labor income data for male household heads from the PSID. I find that the estimate of the variance of the deterministic growth-rate component is zero, while the variance of the shock to the random-walk component is significant and substantial. Hence, the data utilized in this paper favor the RIP model with a permanent random-walk component and a mean-reverting persistent process. I also show, for a PSID-sample comprising the same individuals, that the estimated growth-rate heterogeneity falls if the time dimension of the sample is increased.

[1] I elaborate on this point in Section 2.2.2. Briefly, the concern is about imprecision of higher-order autocovariances of income growth rates in small samples used to evaluate the hypothesis of the absence of growth-rate heterogeneity in idiosyncratic earnings. In my Monte Carlo samples, the higher-order autocovariances of growth rates are indeed imprecise; however, the minimum-distance procedure utilizing *all* autocovariance moments recovers precise estimates of the growth-rate heterogeneity in small samples if the income process contains deterministic growth-rate heterogeneity.

This pattern is inconsistent with the HIP, as the distribution of the growth-rate heterogeneity should be the same for a fixed cross section of individuals, regardless of the time dimension of the sample. This pattern, however, is expected if the true model contains a random-walk component and an econometrician estimates the misspecified HIP.

The results of this paper are important as they contribute to understanding a number of issues. They speak to the economists' choices for modeling of household consumption, savings, and wealth. If the correct model for idiosyncratic labor income is the HIP, one needs to model individuals as sequentially learning about their own labor income profiles to jointly fit the features of consumption and income data. Guvenen (2007) is an example of such a model that explains the profile of consumption inequality observed in U.S. microdata, and the co-movement of the life-cycle profiles of earnings and consumption for households with different levels of schooling. If a substantial variation in incomes is due to permanent and persistent shocks, as is found in this paper, an appropriate model for household choices of consumption, savings, and wealth is an incomplete markets model with uninsurable persistent and/or permanent shocks. Castañeda, Díaz-Giménez, and Ríos-Rull (2003), utilizing such a model, explained the U.S. wealth and earnings inequality; Scholz, Seshadri, and Khitatrakun (2006) explained more than 80% of the 1992 cross-sectional variation of household wealth observed in data from the Health and Retirement Study. Krebs (2003) is an example of a model where permanent idiosyncratic risk, absent in the estimations of the HIP processes but found to be substantial in this and some other papers,[2] reduces economic growth and individual welfare. De Santis (2007) developed a model where log individual consumption is a random walk due to permanent uninsurable idiosyncratic income shocks and showed that such a model can potentially produce large welfare gains from eliminating business cycles.

Further, the idiosyncratic labor income process, which best fits the data utilized in this paper, places restrictions on the models that endogenize labor incomes. A fruitful starting point can be the model in Krebs (2003), where, in equilibrium, permanent shocks to individual human capital translate into permanent shocks to individual labor incomes.

From a policy perspective, it also matters whether the true income process is the HIP or the RIP. If an objective of the policymaker is to reduce consumption inequality and the true idiosyncratic income process is the HIP with a stochastic component of moderate persistence, the policymaker may want to implement policies that subsidize human capital investments by the disadvantaged; self-insurance will be a sufficient shield against the shocks of moderate persistence. If, however, the true income process is the RIP with substantial permanent shocks, an appropriate policy, in addition to the above-mentioned, is to educate the public about risk-sharing instruments provided by credit institutions, stock, and insurance markets.

This paper is also related to recent research that focuses on estimation of the growth-rate heterogeneity and the persistence of the shocks to household incomes using data

[2]See, for example, Carroll and Samwick (1997), Meghir and Pistaferri (2004), and Moffitt and Gottschalk (1995).

on consumption and income, along with a model of household choices of consumption under incomplete insurance markets (see Guvenen and Smith (2008)). There is a certain advantage to using just income moments to extract that information from the data as this approach does not require assumptions on the information households have about the shocks or their unique growth rates, as well as the insurance markets against those shocks.

The rest of the paper is structured as follows. In Section 2, I present a small-sample Monte Carlo study of income processes found in the literature, and introduce the HIP and RIP models. I estimate income processes on simulated data, and also discuss identification of the models containing a random walk and deterministic growth-rate components when data used for estimation are in first differences. In Section 3, I describe the empirical data I use and then present the empirical results. Section 4 concludes.

2. A Monte Carlo study

In this section, I present the income processes estimated in the literature and perform a Monte Carlo study to explore identification of those income processes in small unbalanced panels.

Let the true income process be

$$y_{it} = \alpha_i + \beta_i h_{it} + p_{it} + \tau_{it} + u_{it,me}, \tag{1}$$

$$p_{it} = p_{it-1} + \xi_{it}, \tag{2}$$

$$\tau_{it} = \theta(L)\varepsilon_{it}, \tag{3}$$

where y_{it} is the idiosyncratic log income of individual i with h years of (potential) labor market experience at time t (alternatively, h_{it} may denote individual i's age at time t)[3]; β_i is individual i's growth rate of income; α_i is individual i's initial level of income (a fixed effect that also picks up individual i's levels of permanent and transitory components at the start of his work career—p_{i0} and τ_{i0}, respectively); p_{it} is the permanent stochastic component of income; ξ_{it} is a mean-zero shock to the permanent component; τ_{it} is the transitory stochastic component of income; ε_{it} is a mean-zero shock to the transitory component; $u_{it,me}$ is a mean-zero measurement error; L is the lag operator so that $L^k x_t = x_{t-k}\ \forall k = 0, \pm 1, \pm 2, \dots$; and $\theta(L)$ is a moving average polynomial in L.

The income process outlined in equations (1)–(3) encompasses most of the income processes estimated in the literature.[4] Hause (1980), Lillard and Weiss (1979), and, more recently, Guvenen (2009) estimated the income process that is driven by "deterministic

[3]Statistically, it matters little if h_{it} stands for potential experience or age; the two are highly correlated in the data and increase by 1 year annually. My empirical sample comprises heads of household of ages 25–64; in the Monte Carlo simulations, I assume that households enter the labor market at age 24 and accumulate 1 year of experience when they turn 25. Thus, $h_{it} = 1$ if individual i's age at time t is 25 or if his potential experience is 1 year.

[4]To account for time-varying variances and covariances, most of the studies in the literature, allow for time-varying variances of stochastic (transitory and, if present, permanent) disturbances. This is the strategy I adopt in Section 3.2.

effects" α_i and β_i, an (autoregressive) AR(1) transitory component affected each period by the transitory shock ε_{it}, and measurement error $u_{it,me}$. I label this process the HIP.[5] Meghir and Pistaferri (2004), Carroll (1992), and Carroll and Samwick (1997) are examples of the studies that assume the presence of a random walk and transitory components in idiosyncratic income, but assume away (or present some evidence against) the deterministic idiosyncratic growth-rate component. I label this process the RIP.

An early description of the income process (1)–(3) can be found in Friedman and Kuznets (1954), who suggested that individual incomes can be represented by the sum of permanent, quasi-permanent, and purely transitory components. However, they were not specific on the exact models of the components. Clearly, the purely transitory component can be modeled as a serially uncorrelated shock and the quasi-permanent component can be modeled as an (autoregressive moving average) ARMA(p, q) process. It is less clear what they had in mind with regard to the permanent component. While the time-invariant permanent component can be modeled as a fixed effect, the permanent component that varies over the life cycle can be modeled as a random walk or/and deterministic growth-rate heterogeneity. Muth (1960) suggested that the decomposition of income into a random walk and a purely transitory shock is consistent with Friedman's ideas on the income process (Friedman (1957)). Early papers that estimated income processes, such as Lillard and Weiss (1979) and Hause (1980), were motivated by a simple on-the-job training hypothesis and modeled the permanent component with the deterministic growth-rate heterogeneity. They did not provide reference to the Friedman–Muth decomposition, but Hause (1980) discussed the random-walk alternative and found it inferior to a HIP formulation for his sample of Swedish males. Abowd and Card (1989) and MaCurdy (1982), using higher-order autocovariances of earnings growth rates in PSID data, rejected the growth-rate heterogeneity in earnings. Baker (1997) and Guvenen (2009) argued that such a test has low power and suggested an unrestricted estimation of the income process using the entire autocovariance matrix. I further discuss these results in Section 2.2.2.

2.1 *Simulation details*

My ultimate goal is to determine whether the process containing random-walk, transitory, and deterministic components can be identified empirically in small samples. To this end, I conduct a Monte Carlo study. I simulate data for a number of individuals "observed" for at most 30 periods using the data generating process of equations (1)–(3). I purposefully do not create a balanced panel data set—to mimic the patterns of PSID data, which I later use in empirical analysis. The PSID may contain at most 30 consecutive records on income for each head of household (from the 1968–1997 waves), but

[5]Note that even though, say, Guvenen (2009) did not model the permanent stochastic component of income explicitly, he allowed a root of the autoregressive representation of τ_{it} to be 1. The studies that did not model the permanent component explicitly found that the largest root of the stochastic component is below unity. They interpreted this as the absence of the random-walk component in idiosyncratic labor income, that is, as if $p_{it} = 0$ for all t.

since many heads first enter the labor market and the survey in different years, and because of attrition and nonresponse, they contribute one or more observation(s) on labor income.[6] In each Monte Carlo run, I replicate the PSID sample I use in my empirical analysis in terms of the number of individuals, the number of person-year observations, and the cross-sectional distribution of age in each year.

The details of simulations are as follows. I assume that α_i and β_i are mean-zero, possibly correlated normally distributed fixed effects with which the head is endowed when he enters the labor market. I further assume that ξ_{it} is an independent and identically distributed (i.i.d.) mean-zero shock to the permanent component of income normally distributed with variance equal to σ_ξ^2; that $\varepsilon_{it} \sim$ i.i.d. $N(0, \sigma_\varepsilon^2)$, and that $u_{it,me} \sim$ i.i.d. $N(0, \sigma_{u,me}^2)$, and that τ_{it} is a moving average process of order 1, an autoregressive process of order 1, or an ARMA$(1,1)$ process. I use these particular representations of the transitory component of earnings for the following reasons. First, RIP studies, such as Abowd and Card (1989) and Meghir and Pistaferri (2004), found that the growth rate in male earnings can be represented by a moving average process of order 2, suggesting that the transitory component is a moving average process of order 1. Second, HIP studies, such as Lillard and Weiss (1979) and Guvenen (2009), modeled the transitory component as an autoregressive process of order 1. The estimated AR(1) process is easy to deal with in computational models featuring incomplete insurance markets and agents with uninsurable earnings risk, as argued in Guvenen (2009). Third, a moving average process of order 1 with the moving average parameter of a small magnitude is hard to distinguish from an autoregressive process of order 1.[7] Finally, ARMA$(1,1)$ transitory processes encompass the autocovariance structure of pure AR(1) and MA(1) processes, and have been used, for example, in Baker (1997) and Haider (2001).

In my empirical sample, I observe the age at which each male head first enters the sample and the number of observations he contributes toward the final sample. I restrict my Monte Carlo samples, replicating the number of individuals (1916), the number of person-year observations (29,753), and the age distribution in each year as observed in

[6]The Monte Carlo design assumes that attrition is not related to the permanent or transitory shocks, and idiosyncratic growth rates. Fitzgerald, Gottschalk, and Moffitt (1998), using data from the PSID, found that attrition does not substantially affect the coefficients in a regression of log labor income for male heads on a set of observable variables, such as age, the square of age, and education, used to extract idiosyncratic incomes. Moreover, while they found that attrition is affected by the volatility of individual incomes and income drops, the effects were small so that they concluded that "attrition is mostly noise," unlikely to affect studies using dynamic measures as outcomes. Meghir and Pistaferri (2004) did not find a significant effect of attrition on their estimates of the autoregressive conditional heteroscedasticity (ARCH) effects in the conditional variances of permanent and transitory shocks; Lillard and Panis (1998), also using data from the PSID, found no evidence of selective attrition for white males based on their permanent unobserved components of income. Guided by these considerations, in my Monte Carlo simulations, I assume that attrition is random.

[7]If the true transitory process is $\tau_{it} = (1+\theta L)\varepsilon_{it}$, it can be represented by an infinite order autoregressive process $\tau_{it} = \theta\tau_{it-1} - \theta^2\tau_{it-2} + \theta^3\tau_{it-3} - \cdots + \varepsilon_{it}$ and approximated by $\tau_{it} = \theta\tau_{it-1} + v_{it}$, where $v_{it} = -\theta^2\tau_{it-2} + \theta^3\tau_{it-3} - \cdots + \varepsilon_{it}$. Galbraith and Zinde-Walsh (1994) showed that low-order autoregressive approximations of an MA(1) process—of order 1 up to order 3—with a moving average parameter of 0.5 and less in absolute value performs the best in terms of minimizing the mean squared error.

my PSID sample. As in my PSID data, the simulated data sets comprise individuals who contribute at least 9 consecutive observations on incomes toward the final sample.[8]

For each estimated income model, I report the results based on 100 simulated samples. The models are identified by fitting the theoretical autocovariances to the autocovariances in the simulated data. Estimation is performed using the minimum distance method with the identity weighting matrix.[9] I now turn to estimation results for different simulated income processes.

2.2 *Identification and results from simulated data*

In this section, I present estimation results on simulated data transformed into first differences. I first discuss identification of the processes containing a random-walk component, a transitory component, a deterministic growth-rate component, and measurement error. I present a set of autocovariance moments that can be used to recover the model parameters in large samples. The autocovariance function for the data in first differences can be used to identify the growth-rate heterogeneity and random-walk components if both are present in the data. Permanent shocks contribute only to the diagonal elements of the autocovariance function, that is, the variances, while the growth-rate heterogeneity contributes, in addition, toward all the off-diagonal elements of the autocovariance function. This information can be used to identify all the components as is shown in detail below.

Alternatively, one can use the moments for log incomes in levels to estimate the income process as was done, for example, in Baker (1997) and Guvenen (2009). In real data, however, the results of estimations based on the moments in levels are affected by specification of initial conditions. Estimations in first differences are not likely to depend on initial conditions as was emphasized, for example, in Meghir and Pistaferri (2004) and MaCurdy (1982).

2.2.1 *Identification* In this section, I provide the intuition behind identification of income processes that contain individual-specific growth rates, a permanent random walk, and mean-reverting transitory components when the data used for estimation are in first differences. In the next section, I show that identification carries over to small unbalanced panels using the minimum distance method, which utilizes all the available information in the autocovariance structure of the data. I present identification for income processes with transitory components modeled as AR(1) or MA(1) processes. As mentioned above, those are the transitory processes commonly used in the HIP and RIP studies. Identification can be achieved for the income processes with a more general class of transitory components modeled as ARMA(p, q) processes.

---

[8]The requirement adopted in the literature of having estimation samples with long (and often consecutive) spells of income observations is not necessary for identification of income processes. I use this requirement to be consistent with other studies in the literature such as Meghir and Pistaferri (2004) and Guvenen (2009).

[9]Altonji and Segal (1996) showed that an identity weighting matrix is the best choice for weighting the moments when estimating models of autocovariance structures on microdata with small samples. Most of the papers in the literature, guided by this result, utilize this weighting matrix.

Encompassing model In first differences, the process (1)–(3) is

$$\Delta y_{it} = \beta_i + \xi_{it} + \theta(L)\Delta\varepsilon_{it} + \Delta u_{it,me}, \tag{4}$$

where $\Delta \equiv 1 - L$.

First, assume that the transitory component is a moving average process of order 1, that is, $\tau_{it} = (1 + \theta L)\varepsilon_{it}$.[10] The theoretical autocovariance moments, $\gamma_k = E[\Delta y_{it}\Delta y_{it-k}]$, of this process are

$$\gamma_0 = \sigma_\xi^2 + \sigma_\beta^2 + (1 + (1-\theta)^2 + \theta^2)\sigma_\varepsilon^2 + 2\sigma_{u,me}^2, \tag{5}$$

$$\gamma_1 = \sigma_\beta^2 - (\theta - 1)^2\sigma_\varepsilon^2 - \sigma_{u,me}^2, \tag{6}$$

$$\gamma_2 = \sigma_\beta^2 - \theta\sigma_\varepsilon^2, \tag{7}$$

$$\gamma_k = \sigma_\beta^2, \quad k \geq 3. \tag{8}$$

The empirical variance–covariance matrix contains $\frac{T(T+1)}{2}$ unique moments, where T is the maximum number of income growth rates observed in the sample. The variance of deterministic growth σ_β^2 can be identified from the vector of moments

$$E[\Delta y_{it}\Delta y_{it+k}] = \sigma_\beta^2\mathbf{1}, \quad k = 3, \ldots, T - t, t = 1, \ldots, T - k, \tag{9}$$

where $\mathbf{1}$ is a vector of ones of the row dimension $\frac{(T-3)(T-2)}{2}$. Empirical analogs of the moments γ_0, γ_1, and γ_2 can be further used to identify the other four parameters: σ_ε^2, σ_ξ^2, $\sigma_{u,me}^2$, and θ. The variance of permanent shocks is uniquely identified; to identify the variances of transitory shocks and the moving average coefficient, however, one needs to restrict the variance of measurement error. In particular, the variance of permanent shocks can be identified from the moment

$$\hat{\sigma}_\xi^2 = E\left[\Delta y_{it} \sum_{j=-2}^{j=2} \Delta y_{it+j}\right] - 5\hat{\sigma}_\beta^2,$$

where $\hat{\sigma}_\beta^2$ is estimated using (9).

If $\tau_{it} = (1 - \phi L)^{-1}\varepsilon_{it}$, that is, the transitory component is an AR(1) process, the theoretical autocovariance moments of the income process in first differences are

$$\gamma_0 = \sigma_\xi^2 + \sigma_\beta^2 + \frac{2}{1+\phi}\sigma_\varepsilon^2 + 2\sigma_{u,me}^2, \tag{10}$$

$$\gamma_1 = \sigma_\beta^2 - \frac{1-\phi}{1+\phi}\sigma_\varepsilon^2 - \sigma_{u,me}^2, \tag{11}$$

$$\gamma_k = \sigma_\beta^2 - \phi^{k-1}\frac{1-\phi}{1+\phi}\sigma_\varepsilon^2, \quad k \geq 2. \tag{12}$$

[10]Absent the growth-rate heterogeneity, the income process in first differences is a moving average process of order 2. This is consistent with the results in Abowd and Card (1989) and Meghir and Pistaferri (2004).

Intuitively, σ_β^2 should be identified from higher-order autocovariances—when the contribution of the transitory component toward the autocovariances approaches zero. The parameters for the transitory process can be identified using the set of moments: $\hat{\phi} = \frac{\hat{\gamma}_{k+1}-\hat{\gamma}_k}{\hat{\gamma}_k-\hat{\gamma}_{k-1}}$, $k \geq 3$, and $\hat{\sigma}_\varepsilon^2 = \frac{(\hat{\gamma}_{k+1}-\hat{\gamma}_k)(1+\hat{\phi})}{\hat{\phi}^{k-1}(1-\hat{\phi})^2}$, $k \geq 3$, where $\hat{\gamma}_k = E[\Delta y_{it} \Delta y_{it-k}]$. The variance of growth-rate heterogeneity σ_β^2 can be then identified from the set of moments (12). Empirical analogs of the first two moments, γ_0 and γ_1, can be further used to uniquely identify the remaining model parameters—the variance of permanent shocks and the variance of measurement error.

If individual incomes contain transitory components modeled as an autoregressive moving average process, identification of the model parameters will be similar to identification of the processes containing moving average and autoregressive processes. In particular, the variance of measurement error and the moving average parameter are not separately identified.

Summarizing, if the income process contains individual-specific growth rates and intercepts, a permanent random-walk component, a mean-reverting transitory component, and measurement error, one can identify the variance of permanent shocks and the variance of the deterministic growth-rate heterogeneity using large-sample data. It is not clear, however, if the model parameters can be recovered using small-sample data. For this purpose, I conduct Monte Carlo experiments, the results of which are discussed in Section 2.2.2.

Misspecified HIP What if the true variance of the growth-rate heterogeneity is zero and the income process contains a random-walk component, but an econometrician estimates the HIP model instead?

In a time series context, the asymptotic variance of the scaled mean of a mean-zero stationary process, $\lim_{T\to\infty} E[\frac{1}{\sqrt{T}}\sum_{t=1}^{T} \Delta y_t]^2$, will be equal to $\sum_{k=-\infty}^{\infty} \gamma(k)$ or to the sum of the variance and twice the sum of the nonzero autocovariances if the autocovariances are absolutely summable.[11] The moment succinctly summarizes *all* the information contained in the autocovariance structure of the data. For the income model in equation (4) with $\sigma_\beta^2 = 0$ and the transitory component modeled as an MA(1), the asymptotic variance of the (scaled) sample mean is[12]

$$\lim_{T\to\infty} E\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T} \Delta y_t\right]^2 = \sigma_\xi^2. \tag{13}$$

The empirical analog of equation (13), for a covariance-stationary process, can be estimated from longitudinal data as $\frac{1}{T}[T\gamma_0 + 2(T-1)\gamma_1 + 2(T-2)\gamma_2] = \sigma_\xi^2 + \frac{2}{T}[\sigma_\varepsilon^2(1+\theta^2) + \sigma_{u,me}^2]$.

[11] See, for example, Hamilton (1994, Chap. 7) for a proof. This moment identifies the long-run variance of $\{\Delta y_t\}$. In the context of longitudinal data, γ_k will be zero for $k \geq \min\{T-1, H-1\}$, where H is the maximum age at which an individual can be observed in the data set and T is the time dimension of the data set. In the following, I assume that $H > T$; this assumption is satisfied in the PSID data I use, where $h = 1, \dots, H = 40$ ($h = 1$ corresponds to age 25 and $h = H$ corresponds to age 64) and $T = 30$ (years 1968–1997).

[12] In a time series context, if the transitory income component is an AR(1) or ARMA(1, 1) process, the moment condition (13) will be the same; it will also identify σ_ξ^2 only.

The estimated moment will be closer to σ_ξ^2 for a larger time dimension of the data, T. If, however, the random walk is ignored in estimation, the theoretical autocovariance function is nonzero beyond order 2 and is equal to σ_β^2. The moment in equation (13) will be estimated as $\frac{1}{T}[T\gamma_0 + 2(T-1)\gamma_1 + 2(T-2)\gamma_2 + \cdots + 2\gamma_{T-1}] = \frac{1}{T}\hat{\sigma}_\beta^2[T + 2(T-1) + 2(T-2) + \cdots + 4 + 2] + \frac{2}{T}[\hat{\sigma}_\varepsilon^2(1+\hat{\theta}^2) + \hat{\sigma}_{u,me}^2] = T\hat{\sigma}_\beta^2 + \frac{2}{T}[\hat{\sigma}_\varepsilon^2(1+\hat{\theta}^2) + \hat{\sigma}_{u,me}^2]$, where $\hat{\sigma}_\varepsilon^2$, $\hat{\sigma}_{u,me}^2$, and $\hat{\theta}$ differ from their true values.

The moment estimated in the data, generated by a model that contains a random walk and no growth-rate heterogeneity, should be replicated by the true and the misspecified models. Equating the two moments, it follows that if the true data generating process consists of a random walk, a persistent moving average component, and measurement error, and an econometrician estimates the (misspecified) HIP instead, the variance of the deterministic growth component will be approximately equal to

$$\hat{\sigma}_\beta^2 \approx \frac{1}{T}\sigma_\xi^2. \tag{14}$$

Major microdata sets in the United States have no more than 30 years of consecutive observations on individual labor incomes. Thus, if the true variance of permanent shocks equals 0.02 and $T+1=30$, the variance of the deterministic growth will be estimated at about 0.0007—within the bounds of the typical estimates of the HIP in the literature.

The same logic holds if the transitory stochastic component of income is an AR(1) process. If the true income process is the RIP with a permanent random-walk component, the empirical analog of the moment in equation (13) estimated from longitudinal data is equal to $\frac{1}{T}[T\gamma_0 + 2(T-1)\gamma_1 + 2(T-2)\gamma_2 + \cdots + 2\gamma_{T-1}] = \sigma_\xi^2 + \frac{2\sigma_\varepsilon^2}{1+\phi} - \frac{2}{T}\frac{1-\phi}{1+\phi}\sum_{j=1}^{T-1}\phi^{j-1}(T-j) + \frac{2}{T}\sigma_{u,me}^2$. If the random walk is ignored and the HIP is estimated instead, the moment will be estimated as $T\hat{\sigma}_\beta^2 + \frac{2\hat{\sigma}_\varepsilon^2}{1+\hat{\phi}} - \frac{2}{T}\frac{1-\hat{\phi}}{1+\hat{\phi}}\sum_{j=1}^{T-1}\hat{\phi}^{j-1}(T-j) + \frac{2}{T}\hat{\sigma}_{u,me}^2$.

Thus, one should expect, for any mean-reverting transitory process, that the estimated variance of growth-rate heterogeneity is inversely related to the time dimension of the data set and directly related to the variance of permanent shocks, provided the true income process contains a random-walk component but the estimated process is the (misspecified) HIP.

2.2.2 *Simulation results* In this section, I present the results of estimations of income processes using small-sample, simulated data transformed into first differences.

In Table 1, I list the values for the model parameters used in Monte Carlo experiments.

In Table 2, I estimate the process that contains the individual-specific intercepts and growth rates, a random-walk component, a stationary component modeled as an ARMA(1, 1) process, an AR(1) process or a moving average process of order 1, and measurement error. The true values of the parameters in the models are the variance of fixed effects, $\sigma_\alpha^2 = 0.03$; the variance of the growth-rate heterogeneity, $\sigma_\beta^2 = 0.0004$; the variance of permanent shocks, $\sigma_\xi^2 = 0.02$; the variance of the shocks to the transitory component, $\sigma_\varepsilon^2 = 0.04$; the moving average parameter, $\theta = 0.50$ for a model with the transitory component modeled as an MA(1) process; the autoregressive parameter, $\phi = 0.50$

TABLE 1. Monte Carlo simulations for the income process $y_{it} = \alpha_i + \beta_i h_{it} + p_{it} + \tau_{it} + u_{it,me}$.

	Distribution	Parameter Values
Heterogeneity: $\alpha_i + \beta_i h_{it}$		
α_i	i.i.d. $N(0, \sigma_\alpha^2)$	$\sigma_\alpha^2 = 0.03$
β_i	i.i.d. $N(0, \sigma_\beta^2)$	$\sigma_\beta^2 = 0.0004$
Permanent stochastic component: p_{it}		
$p_{it} = p_{it-1} + \xi_{it}$		
ξ_{it}	i.i.d. $N(0, \sigma_\xi^2)$	$\sigma_\xi^2 = 0.02$
Transitory stochastic component: τ_{it}		
AR(1): $\tau_{it} = \phi\tau_{it-1} + \varepsilon_{it}$		
MA(1): $\tau_{it} = \varepsilon_{it} + \theta\varepsilon_{it-1}$		
ARMA(1, 1): $\tau_{it} = \phi\tau_{it-1} + \varepsilon_{it} + \tilde{\theta}\varepsilon_{it-1}$		
ϕ		0.50
θ		0.50
$\tilde{\theta}$		−0.20
ε_{it}	i.i.d. $N(0, \sigma_\varepsilon^2)$	$\sigma_\varepsilon^2 = 0.04$
Potential experience/age: h_{it}		
$h_{it+1} = h_{it} + 1$		
Measurement error/a purely transitory shock		
$u_{it,me}$	i.i.d. $N(0, \sigma_{u,me}^2)$	$\sigma_{u,me}^2 = 0.02$

TABLE 2. Estimates of the HIP with a random-walk component: simulated data.[a]

Parameters/Trans. Comp.	(1) ARMA(1, 1) $\sigma_\beta^2 = 0.0004$, $\sigma_\xi^2 = 0.02$	(2) AR(1) $\sigma_\beta^2 = 0.0004$, $\sigma_\xi^2 = 0.02$	(3) MA(1) $\sigma_\beta^2 = 0.0004$, $\sigma_\xi^2 = 0.02$
Heterog. growth, $\hat{\sigma}_\beta^2$	0.0004	0.00039	0.0004
	(0.00019)	(0.00017)	(0.0001)
Var. perm. shock, $\hat{\sigma}_\xi^2$	0.019	0.018	0.018
	(0.004)	(0.003)	(0.001)
AR, $\hat{\phi}$	0.472	0.501	—
	(0.156)	(0.065)	—
MA, $\hat{\theta}$	−0.199	—	0.428
	(0.103)	—	(0.013)
$\hat{\sigma}_\varepsilon^2$	0.046	0.042	0.047
	(0.005)	(0.003)	(0.001)
$\sigma_{u,me}^2$	0.015	0.02	0.015
	—	(0.003)	—
Median χ^2 [d.f.]	755.28 [430]	748.65 [430]	825.14 [431]
Rejection rate at 1%	100%	100%	100%

[a]The true income process is $y_{it} = \alpha_i + \beta_i h_{it} + p_{it} + \tau_{it} + u_{it,me}$, with $(1 - L)p_{it+1} = \xi_{it+1}$, $\sigma_\alpha^2 = 0.03$, $\sigma_\beta^2 = 0.0004$, $\sigma_\xi^2 = 0.02$, and $\sigma_{u,me}^2 = 0.02$. In column 1, the transitory process is modeled as $\tau_{it} = \frac{1+\theta L}{1-\phi L}\varepsilon_{it}$, $\phi = 0.50$, $\theta = -0.20$. In column 2, $\tau_{it} = \frac{\varepsilon_{it}}{1-\phi L}$, $\phi = 0.50$. In column 3, $\tau_{it} = (1 + \theta L)\varepsilon_{it}$, $\theta = 0.50$. In all models, the true variance of the shocks to the transitory component is $\sigma_\varepsilon^2 = 0.04$. Prior to estimation, simulated data are transformed to first differences; models are estimated by the equally weighted minimum distance method. Standard errors are given in parentheses and are calculated as the standard deviations of the estimates across 100 model simulations. The goodness-of-fit statistic is based on Newey (1985).

for a model with the transitory component modeled as an AR(1) process and $\phi = 0.50$, $\theta = -0.20$ for a model with the transitory component modeled as an ARMA(1, 1) process; and the variance of measurement error, $\sigma^2_{u,me} = 0.02$.[13] The chosen values for the true parameters are within the range of the values estimated in the literature.[14] An individual with a (deterministic) growth rate 1 standard deviation above the mean will have a 2% advantage in income every year relative to an observationally equivalent individual whose income does not grow. A 1 standard deviation in the permanent shock translates into a permanent change in income of about 14%, while a 1 standard deviation in the transitory shock causes income to change by 20% in the current period. Note that when the transitory process contains a moving average component—columns 1 and 3—the variance of measurement error is not identified.

Regardless of the model for the transitory component, the variance of growth-rate heterogeneity σ^2_β and the variance of permanent shocks σ^2_ξ are recovered without any biases and statistically precisely by the equally weighted minimum distance method.[15] In columns 1 and 3, I set the variance of measurement error to the value that equals 25% of the variance of income growth rates.[16] The ratio of the assumed to the true variance of measurement error is about 75%. As a result, in column 1, the autoregressive persistence is estimated at a value close to its true value of 0.50, while the estimated variance of transitory shocks is somewhat larger than its true value. It is possible to identify all the parameters when the transitory component is an autoregressive process of order 1, which is confirmed in column 2. In column 3, the transitory component is modeled as an MA(1) process. Since the variance of measurement error set in estimations differs from its true value, the estimated moving average parameter is somewhat below its true value, while the estimated variance of the shocks to the transitory process is slightly above its true value.[17] The last two rows in Table 2 report the median value of the goodness-of-fit statistics across 100 estimations and the frequency of model rejections at the 1% significance level. The result in the last row is quite important. The size of the χ^2 test is severely distorted: instead of rejecting the true model 1 time out of 100, the test rejects the true model 100% of the time. It appears that the χ^2 test of the model validity is not likely to be useful in empirical applications utilizing unbalanced small-sample panel data.

[13]The results presented below are qualitatively similar when more or less persistent transitory processes are chosen in simulations.

[14]The variance of fixed effects may seem low, but is comparable to the estimates in Guvenen (2009) and is inconsequential for the results as my focus is on estimation in first differences. The variance of measurement error implies, for example, for the income process with τ_{it} modeled as an AR(1) process, that 35% of the variation in observed income growth rates is due to measurement error.

[15]I also verified that the results are robust to other choices for the variance of fixed effects. Table 3 shows that the estimates are largely unaffected when the variance of fixed effects is set to 0.10, the value which is more than three times larger than its value in the original simulations. It is not surprising as the fixed effects are canceled out by first differencing and, therefore, do not effect the moments used for estimation.

[16]This is consistent with Meghir and Pistaferri (2004) and findings in the literature on measurement error in longitudinal income data surveyed by Bound, Brown, and Mathiowetz (2001).

[17]If the variance of measurement error was set to its true value in estimations of the models in columns 1 and 3, all the model parameters would be estimated without any biases. The results are not reported for brevity.

Table 3. Estimates of the HIP with a random-walk component and large variance of fixed effects: simulated data.[a]

Parameters/Trans. Comp.	(1) ARMA(1, 1) $\sigma_\beta^2 = 0.0004$, $\sigma_\xi^2 = 0.02$	(2) AR(1) $\sigma_\beta^2 = 0.0004$, $\sigma_\xi^2 = 0.02$	(3) MA(1) $\sigma_\beta^2 = 0.0004$, $\sigma_\xi^2 = 0.02$
Heterog. growth, $\hat{\sigma}_\beta^2$	0.00037	0.0004	0.0004
	(0.00015)	(0.00018)	(0.00013)
Var. perm. shock, $\hat{\sigma}_\xi^2$	0.019	0.018	0.019
	(0.003)	(0.0035)	(0.002)
AR, $\hat{\phi}$	0.423	0.512	—
	(0.200)	(0.071)	—
MA, $\hat{\theta}$	−0.167	—	0.428
	(0.143)	—	(0.012)
$\hat{\sigma}_\varepsilon^2$	0.045	0.041	0.047
	(0.005)	(0.004)	(0.001)
$\sigma_{u,me}^2$	0.015	0.019	0.015
	—	(0.003)	—
Median χ^2 [d.f.]	762.96 [430]	759.89 [430]	868.12 [431]
Rejection rate at 1%	100%	100%	100%

[a]The true income process is $y_{it} = \alpha_i + \beta_i h_{it} + p_{it} + \tau_{it} + u_{it,me}$, with $(1 - L)p_{it+1} = \xi_{it+1}$, $\sigma_\alpha^2 = 0.10$, $\sigma_\beta^2 = 0.0004$, $\sigma_\xi^2 = 0.02$, and $\sigma_{u,me}^2 = 0.02$. In column 1, the transitory process is modeled as $\tau_{it} = \frac{1+\theta L}{1-\phi L}\varepsilon_{it}$, $\phi = 0.50$, $\theta = -0.20$. In column 2, $\tau_{it} = \frac{\varepsilon_{it}}{1-\phi L}$, $\phi = 0.50$. In column 3, $\tau_{it} = (1 + \theta L)\varepsilon_{it}$, $\theta = 0.50$. In all models, the true variance of the shocks to the transitory component is $\sigma_\varepsilon^2 = 0.04$. Prior to estimation, simulated data are transformed to first differences; models are estimated by the equally weighted minimum distance method. Standard errors in parentheses calculated as the standard deviations of the estimates across 100 model simulations. The goodness-of-fit statistic is based on Newey (1985).

Guvenen (2009), in a simulation exercise, showed that the tests of higher-order autocovariances equal to zero falsely reject the growth-rate heterogeneity even when the true income process contains idiosyncratic growth rates. This test was previously used by MaCurdy (1982). Table 4 confirms this result. I first create 100 samples generated in accordance with the models in Table 2, which contain deterministic idiosyncratic growth rates, a permanent random-walk component, and a transitory stochastic process. For each simulated sample, I calculate the empirical autocovariance function. The results in the columns are the averages of the autocovariances of a given order across 100 simulated samples; standard errors, given in parentheses, are calculated as the standard deviations of these estimates across 100 simulated samples. In column 1, the true process contains an ARMA(1, 1) component; in column 2, an AR(1) component; in column 3, an MA(1) component. As can be seen from column 3, only autocovariances of orders 0, 1, and 2 are significant. The rest are insignificant, even though the magnitude of the autocovariances of orders 3 and higher are positive and some approach the true variance of the deterministic growth-rate heterogeneity. For the model with AR(1) or ARMA(1, 1) transitory processes, the autocovariance function is significant only from order 0 to order 3, inclusive, and the contribution of the transitory component toward the autocovariance function dissipates rather quickly. The minimum distance procedure, however, uses the entire autocovariance function, not only the information contained in higher-

TABLE 4. Autocovariances for income growth rates for income processes with growth-rate heterogeneity and a random-walk component: simulated data.[a]

Order	(1) $\tau_{it} \sim$ ARMA(1, 1)	(2) $\tau_{it} \sim$ AR(1)	(3) $\tau_{it} \sim$ MA(1)
0	0.12066	0.11338	0.1196
	(0.00108)	(0.0011)	(0.001)
1	−0.04289	−0.03323	−0.02989
	(0.00089)	(0.00079)	(0.00078)
2	−0.00335	−0.00639	−0.01964
	(0.00084)	(0.0007)	(0.00079)
3	−0.00137	−0.00307	0.00038
	(0.00078)	(0.00067)	(0.00089)
4	−0.000528	−0.00136	0.00054
	(0.00096)	(0.0007)	(0.00092)
5	−0.00016	−0.0005	0.00037
	(0.00104)	(0.0009)	(0.00091)
10	0.00033	0.00057	0.00055
	(0.00123)	(0.0011)	(0.0012)
15	0.00028	0.00048	0.00036
	(0.0016)	(0.0016)	(0.00188)
20	0.00028	0.0003	0.00034
	(0.0029)	(0.0031)	(0.00302)

[a]The true income process is $y_{it} = \alpha_i + \beta_i h_{it} + p_{it} + \tau_{it} + u_{it,me}$, with $(1-L)p_{it+1} = \xi_{it+1}$, $\sigma_\alpha^2 = 0.03$, $\sigma_\beta^2 = 0.0004$, $\sigma_\xi^2 = 0.02$, and $\sigma_{u,me}^2 = 0.02$. In column 1, $\tau_{it} = \frac{1+\theta L}{1-\phi L}\varepsilon_{it}$, $\phi = 0.50$, $\theta = -0.20$. In column 2, $\tau_{it} = \frac{\varepsilon_{it}}{1-\phi L}$, $\phi = 0.50$. In column 3, the transitory process is modeled as $\tau_{it} = (1+\theta L)\varepsilon_{it}$, $\theta = 0.50$. The true variance of the shocks to the transitory component is $\sigma_\varepsilon^2 = 0.04$. Simulated data are transformed to first differences. Autocovariances of a given order are the averages of the autocovariances in simulated data across 100 simulations. Standard errors are given in parentheses and are calculated as the standard deviations of the estimated autocovariances of a given order across 100 model simulations.

order autocovariances, and its sample variability to uncover correctly and precisely the variance of the deterministic growth-rate heterogeneity—Table 2, columns 1–3. To summarize, the test for detecting the random-growth component using higher-order autocovariances lacks power and is of little practical guidance when using small-sample unbalanced data as is typical in the literature. The random growth component, however, can be recovered if one estimates the model utilizing the entire autocovariance function of income growth rates.

In Table 5, the true income process contains the individual-specific intercept, a random-walk component, a transitory component, and measurement error. The true variance of the random-walk shock equals 0.02, while the true variance of deterministic growth-rate heterogeneity equals zero. The process is estimated as the (misspecified) HIP containing a deterministic growth-rate component and an unrestricted ARMA(1, 1) process (column 1), AR(1) process (column 2), or MA(1) process (column 3). The variance of the shock to the transitory component is estimated at about 0.05 in columns 2 and 3, and at 0.07 in column 1, while the estimated values of the autoregressive parameter are substantially biased upward (columns 1 and 2). Importantly, when the random-walk component is ignored in estimation, the long-run persistence of the process is captured instead by the variance of the deterministic growth, with the estimated value sub-

TABLE 5. Estimates of the misspecified HIP: simulated data.[a]

Parameters/Trans. Comp.	(1) ARMA(1, 1) $\sigma^2_\beta = 0$, $\sigma^2_\xi = 0.02$	(2) AR(1) $\sigma^2_\beta = 0$, $\sigma^2_\xi = 0.02$	(3) MA(1) $\sigma^2_\beta = 0$, $\sigma^2_\xi = 0.02$
Heterog. growth, $\hat{\sigma}^2_\beta$	0.0005	0.00052	0.0007
	(0.00009)	(0.00008)	(0.00009)
Var. perm. shock, $\hat{\sigma}^2_\xi$	0.00	0.00	0.00
	—	—	—
AR, $\hat{\phi}$	0.762	0.675	—
	(0.044)	(0.032)	—
MA, $\hat{\theta}$	−0.261	—	0.419
	(0.029)	—	(0.009)
$\hat{\sigma}^2_\varepsilon$	0.069	0.054	0.058
	(0.001)	(0.002)	(0.0005)
$\sigma^2_{u,me}$	0.015	0.024	0.015
	—	(0.002)	—
Median χ^2 [d.f.]	867.15 [431]	921.86 [431]	1488.38 [432]
Rejection rate at 1%	100%	100%	100%

[a]The true income process is $y_{it} = \alpha_i + p_{it} + \tau_{it} + u_{it,me}$, with $(1 - L)p_{it+1} = \xi_{it+1}$, $\sigma^2_\alpha = 0.03$, $\sigma^2_\xi = 0.02$, and $\sigma^2_{u,me} = 0.02$. In column 1, the transitory process is modeled as $\tau_{it} = \frac{1+\theta L}{1-\phi L}\varepsilon_{it}$, $\phi = 0.50$, $\theta = -0.20$. In column 2, $\tau_{it} = \frac{\varepsilon_{it}}{1-\phi L}$, $\phi = 0.50$. In column 3, $\tau_{it} = (1 + \theta L)\varepsilon_{it}$, $\theta = 0.50$. In all models, the true variance of the shocks to the transitory component is $\sigma^2_\varepsilon = 0.04$. Prior to estimation, simulated data are transformed to first differences; models are estimated by the equally weighted minimum distance method. Standard errors are given in parentheses and are calculated as the standard deviations of the estimates across 100 model simulations. The goodness-of-fit statistic is based on Newey (1985).

stantially and significantly away from its true value of zero. When the model contains a moving average transitory process, this is the estimate one can expect given the time series dimension of 29 periods for income growth rates (see Section 2.2.1).

Baker (1997) and Guvenen (2009) estimated models similar to those in Table 5 and considered such specifications as nesting the random-walk and the growth-rate heterogeneity hypotheses. They conjectured that the unit-root hypothesis could be rejected if the estimated autoregressive persistence $\hat{\phi}$ was below 1 and tightly estimated. The models of Table 5, however, do not properly encompass the income process with a random walk, a persistent component, and growth-rate heterogeneity. If, for example, a time series is composed of a random walk and an autoregressive component with low or moderate persistence ϕ, the resulting model with one disturbance is an ARMA(1, 1) process in first differences with autoregressive persistence ϕ.[18] Baker (1997) estimated such a model using PSID data and found low autoregressive persistence and significant variance of the random-growth component. The results in Table 5 indicate that such models are not proper nesting specifications, and can result in significant estimates of the growth-rate heterogeneity and in low to moderate autoregressive persistence—as was

[18]If $y_{it} = p_{it} + \tau_{it}$, where $p_{it} = p_{it-1} + \xi_{it}$ and $\tau = \phi\tau_{it-1} + \varepsilon_{it}$, the income process can be written as $(1-\phi L)\Delta y_{it} = (1-\phi L)\xi_{it} + (1-L)\varepsilon_{it} = (1+\theta L)u_{it}$, where the moving average parameter θ and the variance of u_{it} depend on ϕ and the variances of the original shocks ξ_{it} and ε_{it}.

Table 6. Estimates of the HIP with no random-walk component: simulated data.[a]

Parameters/Trans. Comp.	ARMA(1, 1) $\sigma^2_\beta = 0.0004$, $\sigma^2_\xi = 0$	AR(1) $\sigma^2_\beta = 0.0004$, $\sigma^2_\xi = 0$	MA(1) $\sigma^2_\beta = 0.0004$, $\sigma^2_\xi = 0$
Heterog. growth, $\hat{\sigma}^2_\beta$	0.00035	0.00035	0.00035
	(0.00005)	(0.00006)	(0.00005)
Var. perm. shock, $\hat{\sigma}^2_\xi$	0.0004	0.00029	0.00008
	(0.0012)	(0.0007)	(0.0003)
AR, $\hat{\phi}$	0.463	0.479	—
	(0.135)	(0.042)	—
MA, $\hat{\theta}$	−0.225	—	0.402
	(0.105)	—	(0.01)
$\hat{\sigma}^2_\varepsilon$	0.047	0.04	0.049
	(0.002)	(0.002)	(0.0006)
$\sigma^2_{u,me}$	0.013	0.02	0.013
	—	(0.003)	—
Median χ^2 [d.f.]	803.84 [430]	745.10 [430]	816.09 [431]
Rejection rate at 1%	100%	100%	100%

[a]The true income process is $y_{it} = \alpha_i + \beta_i h_{it} + \tau_{it} + u_{it,me}$, with $\sigma^2_\alpha = 0.03$, $\sigma^2_\beta = 0.0004$, and $\sigma^2_{u,me} = 0.02$. In the second column, the transitory process is modeled as $\tau_{it} = \frac{1+\theta L}{1-\phi L}\varepsilon_{it}$, $\phi = 0.50$, $\theta = -0.20$. In the third column, $\tau_{it} = \frac{\varepsilon_{it}}{1-\phi L}$, $\phi = 0.50$. In the last column, $\tau_{it} = (1+\theta L)\varepsilon_{it}$, $\theta = 0.50$. In all models, the true variance of the shocks to the transitory component is $\sigma^2_\varepsilon = 0.04$. Prior to estimation, simulated data are transformed to first differences; models are estimated by the equally weighted minimum distance method. Standard errors are given in parentheses and are calculated as the standard deviations of the estimates across 100 model simulations. The goodness-of-fit statistic is based on Newey (1985).

found in Baker (1997) and Guvenen (2009)—when the true model contains a random-walk component, a transitory component with low persistence, and no growth-rate heterogeneity.

Other experiments For completeness, I also considered models that contain a transitory component and idiosyncratic trends but no random-walk component, while in estimations I allowed for a permanent random-walk component (Table 6). Briefly, in Table 6, the variance of growth-rate heterogeneity is precisely recovered in estimations, while the variance of permanent shocks is small in magnitude and not statistically different from its true value of zero. This result has important implications for empirical analysis: if incomes differ over the life cycle due to deterministic growth-rate heterogeneity and are not affected by the shocks that persist over the entire life cycle, one can expect that the model that allows for both components will recover the true variance of growth-rate heterogeneity and a small and imprecise variance of the random-walk shocks.

I also considered models with a random-walk component but no growth-rate heterogeneity, while in estimations I allowed for both components. The results are in Table 7: while the variance of random-walk shocks is tightly estimated, the variance of growth-rate heterogeneity is numerically small and insignificant.

Table 8 shows the estimates of misspecified RIP processes when the true income processes contain deterministic growth-rate heterogeneity while the estimated models contain a random-walk component and no idiosyncratic growth rates. The estimated

TABLE 7. Estimates of the RIP with no growth-rate heterogeneity: simulated data.[a]

Parameters/Trans. Comp.	ARMA(1, 1) $\sigma^2_\beta = 0$, $\sigma^2_\xi = 0.02$	AR(1) $\sigma^2_\beta = 0$, $\sigma^2_\xi = 0.02$	MA(1) $\sigma^2_\beta = 0$, $\sigma^2_\xi = 0.02$
Heterog. growth, $\hat{\sigma}^2_\beta$	0.00006	0.00005	0.00004
	(0.0001)	(0.00007)	(0.00006)
Var. perm. shock, $\hat{\sigma}^2_\xi$	0.018	0.018	0.019
	(0.003)	(0.002)	(0.001)
AR, $\hat{\phi}$	0.488	0.510	—
	(0.169)	(0.062)	—
MA, $\hat{\theta}$	−0.204	—	0.495
	(0.117)	—	(0.014)
$\hat{\sigma}^2_\varepsilon$	0.047	0.041	0.04
	(0.004)	(0.003)	(0.001)
$\sigma^2_{u,me}$	0.015	0.02	0.013
	—	(0.004)	—
Median χ^2 [d.f.]	828.90 [430]	862.06 [430]	909.98 [431]
Rejection rate at 1%	100%	100%	100%

[a]The true income process is $y_{it} = \alpha_i + p_{it} + \tau_{it} + u_{it,me}$, with $\sigma^2_\alpha = 0.03$, $\sigma^2_\xi = 0.02$, and $\sigma^2_{u,me} = 0.02$. In the second column, the transitory process is modeled as $\tau_{it} = \frac{1+\theta L}{1-\phi L}\varepsilon_{it}$, $\phi = 0.50$, $\theta = -0.20$. In the third column, $\tau_{it} = \frac{\varepsilon_{it}}{1-\phi L}$, $\phi = 0.50$. In the last column, $\tau_{it} = (1+\theta L)\varepsilon_{it}$, $\theta = 0.50$. In all models, the true variance of the shocks to the transitory component is $\sigma^2_\varepsilon = 0.04$. Prior to estimation, simulated data are transformed to first differences; models are estimated by the equally weighted minimum distance method. Standard errors are given in parentheses and are calculated as the standard deviations of the estimates across 100 model simulations. The goodness-of-fit statistic is based on Newey (1985).

variance of permanent shocks is nonnegligible and significant when the transitory component is an AR(1) or an ARMA(1, 1) process—see the second and third columns—but small and insignificant when the transitory component is an MA(1) process (last column).

The combined results of Tables 2, 5, 6, 7, and 8 highlight the importance of estimating an encompassing process to guard against misspecified estimates of either the random-walk or growth-rate heterogeneity components.

Table 9 extends the results of Table 2, allowing for estimation of the persistence of the permanent component. As in a time series context, the estimated persistence is biased downward yet close to unity; the estimated variance of growth-rate heterogeneity is slightly upward-biased, but still statistically significant. The results confirm that testing for a unit root is challenging not only using short time series data, but also using small unbalanced longitudinal data. Table 10 extends the results of Table 2, modeling the permanent component as an AR(1) process with persistence equal to 0.95. Relative to the results in Table 9, the estimated persistence of the permanent component is downward-biased; the parameters of the transitory process are estimated with less precision, and the estimated variance of growth-rate heterogeneity is biased downward and is less precise. It appears that estimation of the income process (1)–(3) in small unbalanced samples will recover the true parameters reasonably well when the true persistence of the permanent component is 1 or very close to unity; estimations will perform relatively worse when the permanent component is less persistent.

Table 8. Estimates of the misspecified RIP: simulated data.[a]

Parameters/Trans. Comp.	ARMA(1, 1) $\sigma^2_\beta = 0.0004$, $\sigma^2_\xi = 0$	AR(1) $\sigma^2_\beta = 0.0004$, $\sigma^2_\xi = 0$	MA(1) $\sigma^2_\beta = 0.0004$, $\sigma^2_\xi = 0$
Heterog. growth, $\hat{\sigma}^2_\beta$	0.00	0.00	0.00
	—	—	—
Var. perm. shock, $\hat{\sigma}^2_\xi$	0.008	0.008	0.0006
	(0.001)	(0.001)	(0.0008)
AR, $\hat{\phi}$	0.712	0.749	—
	(0.049)	(0.030)	—
MA, $\hat{\theta}$	−0.283	—	0.607
	(0.037)	—	(0.011)
$\hat{\sigma}^2_\varepsilon$	0.037	0.033	0.052
	(0.002)	(0.002)	(0.0008)
$\sigma^2_{u,me}$	0.015	0.02	0.013
	—	(0.001)	—
Median χ^2 [d.f.]	764.58 [431]	763.68 [431]	994.70 [432]
Rejection rate at 1%	100%	100%	100%

[a]The true income process is $y_{it} = \alpha_i + \beta_i h_{it} + \tau_{it} + u_{it,me}$, with $\sigma^2_\alpha = 0.03$, $\sigma^2_\beta = 0.0004$, and $\sigma^2_{u,me} = 0.02$. In the second column, the transitory process is modeled as $\tau_{it} = \frac{1+\theta L}{1-\phi L}\varepsilon_{it}$, $\phi = 0.50$, $\theta = -0.20$. In the third column, $\tau_{it} = \frac{\varepsilon_{it}}{1-\phi L}$, $\phi = 0.50$. In the last column, $\tau_{it} = (1+\theta L)\varepsilon_{it}$, $\theta = 0.50$. In all models, the true variance of the shocks to the transitory component is $\sigma^2_\varepsilon = 0.04$. Prior to estimation, simulated data are transformed to first differences; models are estimated by the equally weighted minimum distance method. Standard errors are given in parentheses and are calculated as the standard deviations of the estimates across 100 model simulations. The goodness-of-fit statistic is based on Newey (1985).

3. Empirical results

In this section, I estimate time series processes for idiosyncratic labor incomes of male household heads from the PSID. I first describe the data I utilize.

3.1 *Data*

I use income and demographic data from the 1968–1997 waves of the PSID. I select male household heads of ages 25–64.[19] I further drop heads with inconsistent education records and define two education groups. The first group comprises heads who dropped out of high school or just finished high school. The second group includes heads who finished some college, graduated from college, or attained a graduate degree.[20]

The measure of income utilized is the head's labor income from all sources, inclusive of the labor part of farm and business income. Income data in the PSID refer to the previous calendar year; I adjust the data appropriately by the consumer price index for all items normalized to 100 in 1982–1984. I set income observations to missing when

[19]Age in the PSID does not necessarily change in adjacent surveys since information can be collected at different months of a year. Also, some individuals have inconsistent age series which, among other things, may reflect typing errors by interviewers. I utilize information on the year of birth to construct a cleaner measure of age for those heads who have this information in the individual file. Otherwise, I use an individual's age at the time he first appears as a head in the survey to impute his age in other years.

[20]Each education group roughly comprises 50% of the sample.

Table 9. Estimates of the HIP with random-walk component: simulated data.[a]

Parameters/Trans. Comp.	ARMA(1, 1) $\sigma^2_\beta = 0.0004$, $\sigma^2_\xi = 0.02$	AR(1) $\sigma^2_\beta = 0.0004$, $\sigma^2_\xi = 0.02$	MA(1) $\sigma^2_\beta = 0.0004$, $\sigma^2_\xi = 0.02$
Heterog. growth, $\hat{\sigma}^2_\beta$	0.00056	0.0005	0.00056
	(0.00027)	(0.00026)	(0.00024)
AR, $\hat{\phi}_p$	0.964	0.97	0.973
	(0.045)	(0.044)	(0.035)
Var. perm. shock, $\hat{\sigma}^2_\xi$	0.021	0.02	0.018
	(0.003)	(0.004)	(0.002)
AR, $\hat{\phi}_\tau$	0.402	0.466	—
	(0.148)	(0.074)	—
MA, $\hat{\theta}$	−0.177	—	0.428
	(0.098)	—	(0.013)
$\hat{\sigma}^2_\varepsilon$	0.042	0.04	0.046
	(0.005)	(0.005)	(0.001)
$\sigma^2_{u,me}$	0.015	0.019	0.013
	—	(0.004)	—
Median χ^2 [d.f.]	752.24 [429]	746.54 [429]	874.26 [430]
Rejection rate at 1%	100%	100%	100%

[a]The true income process is $y_{it} = \alpha_i + \beta_i h_{it} + p_{it} + \tau_{it} + u_{it,me}$, with $\sigma^2_\alpha = 0.03$, $\sigma^2_\beta = 0.0004$, and $\sigma^2_{u,me} = 0.02$. In the second column, the transitory process is modeled as $\tau_{it} = \frac{1+\theta L}{1-\phi_\tau L}\varepsilon_{it}$, $\phi_\tau = 0.50$, $\theta = -0.20$. In the third column, $\tau_{it} = \frac{\varepsilon_{it}}{1-\phi_\tau L}$, $\phi_\tau = 0.50$. In the last column, $\tau_{it} = (1+\theta L)\varepsilon_{it}$, $\theta = 0.50$. $p_{it} = \phi_p p_{it-1} + \xi_{it}$, where $\phi_p = 1$ and $\sigma^2_\xi = 0.02$. In all models, the true variance of the shocks to the transitory component is $\sigma^2_\varepsilon = 0.04$. Prior to estimation, simulated data are transformed to first differences; models are estimated by the equally weighted minimum distance method. Standard errors are given in parentheses and are calculated as the standard deviations of the estimates across 100 model simulations. The goodness-of-fit statistic is based on Newey (1985).

the head reports being a student or self-employed and in the year subsequent to that report. When the head reports being retired, I set his income in that year and all subsequent years to missing. I further drop observations for the years when the percentage change of real labor income in adjacent years is above 500 or below −80. I then drop observations with zero, top-coded, and missing incomes, and select the longest consecutive spell of positive incomes with at least 9 observations. I exclude data for households from the Survey of Economic Opportunity (SEO) subsample, which oversamples the poor. Figure 1 plots the time profile of the variances of log labor income for different PSID samples: the core subsample, the SEO subsample, and the sample comprising those two subsamples. The variances in the core sample—which was representative of the U.S. population in 1968, at the start of the survey—displayed a sharp increase in the beginning of the 1980s. The variances for the SEO subsample follow a somewhat different time pattern: the variance increased in the beginning of the 1980s, but started declining afterward. For this reason, and following most of the literature, I exclude the SEO subsample from my empirical analysis.[21] The final sample contains information for

[21]The pattern of the variances in the sample comprising both the SEO and core subsamples is similar to the pattern of variances in Figure 2 of Meghir and Pistaferri (2004) for their whole sample.

Table 10. Estimates of the RIP with growth-rate heterogeneity: simulated data.[a]

Parameters/Trans. Comp.	ARMA(1, 1)	AR(1)	MA(1)
Heterog. growth, $\hat{\sigma}^2_\beta$	0.0003	0.00026	0.00025
	(0.00018)	(0.0002)	(0.00017)
AR, $\hat{\phi}_p$	0.909	0.926	0.938
	(0.065)	(0.063)	(0.04)
Var. perm. shock, $\hat{\sigma}^2_\xi$	0.024	0.023	0.019
	(0.006)	(0.007)	(0.002)
AR, $\hat{\phi}_\tau$	0.322	0.429	—
	(0.248)	(0.128)	—
MA, $\hat{\theta}$	−0.135	—	0.426
	(0.184)	—	(0.013)
$\hat{\sigma}^2_\varepsilon$	0.040	0.04	0.046
	(0.007)	(0.006)	(0.002)
$\sigma^2_{u,me}$	0.015	0.017	0.015
	—	(0.007)	—
Median χ^2 [d.f.]	752.35 [429]	742.29 [429]	826.40 [430]
Rejection rate at 1%	100%	100%	100%

[a]The true income process is $y_{it} = \alpha_i + \beta_i h_{it} + p_{it} + \tau_{it} + u_{it,me}$, with $\sigma^2_\alpha = 0.03$, $\sigma^2_\beta = 0.0004$, and $\sigma^2_{u,me} = 0.02$. In the second column, the transitory process is modeled as $\tau_{it} = \frac{1+\theta L}{1-\phi_\tau L}\varepsilon_{it}$, $\phi_\tau = 0.50$, $\theta = -0.20$. In the third column, $\tau_{it} = \frac{\varepsilon_{it}}{1-\phi_\tau L}$, $\phi_\tau = 0.50$. In the last column, $\tau_{it} = (1 + \theta L)\varepsilon_{it}$, $\theta = 0.50$. $p_{it} = \phi_p p_{it-1} + \xi_{it}$, where $\phi_p = 0.95$ and $\sigma^2_\xi = 0.02$. Prior to estimation, simulated data are transformed to first differences; models are estimated by the equally weighted minimum distance method. Standard errors are given in parentheses and are calculated as the standard deviations of the estimates across 100 model simulations. The goodness-of-fit statistic is based on Newey (1985).

1916 heads with 29,753 person-year observations on labor incomes. Table 11 contains some descriptive sample statistics for selected years of the sample.

The measure of the idiosyncratic head's labor income growth in each year is the head's residual from a cross-sectional regression of the first difference in log labor income on a third polynomial in age, "college" dummy, and interactions between the "college" dummy and the age polynomial.[22] This regression, which specifies the deterministic component of incomes common to all heads, is similar to specifications adopted in the literature and assumes that returns to the head's experience and education are affected by the aggregate state of the economy, that is, they differ by year.

Table 12 contains the results of the tests of the autocovariances of a given order being zero in all time periods. One cannot reject the null that the autocovariances of orders 4 and 5 are equal to zero. However, the null that the autocovariances of order 3 and higher or order 4 and higher are all equal to zero can be rejected. The results can be consistent with a model containing an AR(1) or ARMA(1, 1) transitory component with a small autoregressive persistence.[23]

[22]The "college" dummy equals 1 if the head finished some college, graduated from college, or attained a graduate degree.

[23]The results are somewhat different from Meghir and Pistaferri (2004), who found, for their pooled sample, that the autocovariances of orders 3 and 4 are not statistically different from zero, and a p-value of 12%

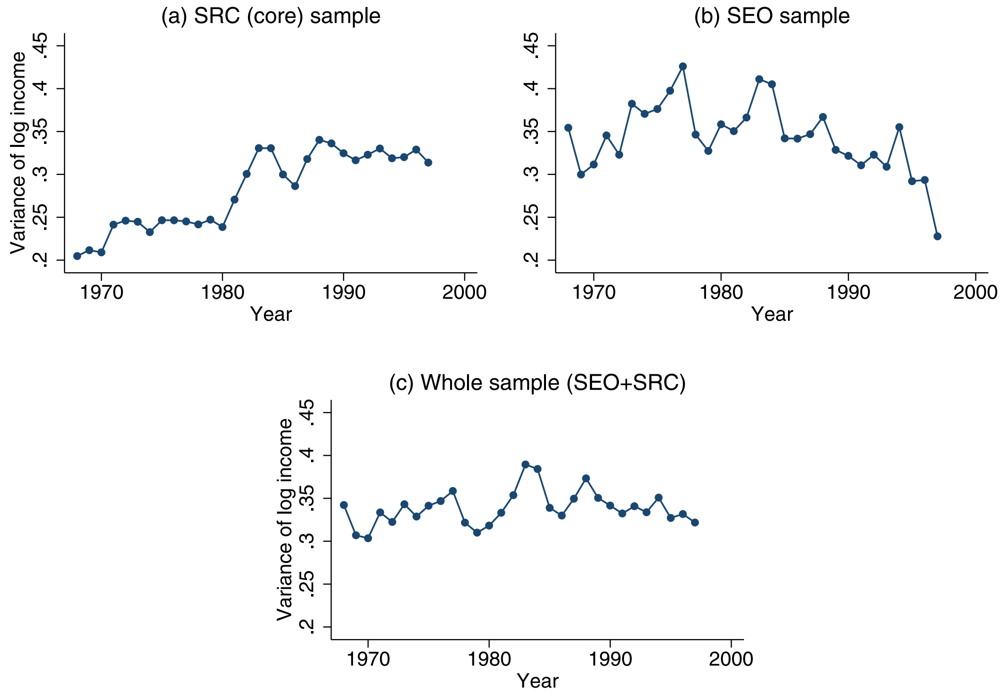

FIGURE 1. The variance of log labor income by year. In panel (a), the graph depicts the variances of log labor income for the main sample that includes heads from the core subsample only. In panel (b), the graph depicts the variances of log labor income for a sample that includes heads from the SEO subsample only. In panel (c), the graph depicts the variances of log labor income for a sample that includes heads from the core and the SEO subsamples. All samples include heads with consecutive spells of at least 9 income observations only.

3.2 *Results*

Table 13 contains the main results.[24] The models are estimated by fitting the empirical autocovariance function to the theoretical autocovariance function utilizing the identity weighting matrix, that is, by the equally weighted minimum distance method.

In column 1, I estimate the HIP process, which ignores the potentially important random-walk component in idiosyncratic labor incomes. The variance of individual-specific growth rates is estimated at 0.0004, which is significant at the 1% level, while the persistence of the transitory component is moderate: the autoregressive parameter is estimated at about 0.70.[25]

for the test that the autocovariances of order 3 and higher are all equal to zero. Their results are based on PSID data up to 1993 and a sample that includes the SEO subsample. If I ignore the data after 1993, my results for the tests in Table 12 are similar to those in Table II in Meghir and Pistaferri (2004).

[24]Most of the studies in the literature allow for time-dependent variances of permanent and/or persistent shocks. My estimates of these parameters in Table 13, columns 1–4 and 6, should be interpreted as the unconditional variances of transitory and permanent shocks.

[25]The autoregressive parameter is estimated at about the same value if the transitory component is modeled as an AR(1) process. I choose to report the results for a model with an ARMA(1, 1) transitory compo-

Table 11. Sample statistics for selected years.

	Year			
	1970	1980	1990	1997
Age	40.67	41.11	39.71	44.37
	(8.50)	(10.83)	(9.06)	(7.46)
Hours	2258	2224	2270	2281
	(460)	(496)	(498)	(495)
Nonfinancial income[a]	33,127	36,289	39,371	45,193
	(13,851)	(15,901)	(24,914)	(27,340)
White	0.89	0.91	0.93	0.93
Married	0.96	0.90	0.87	0.88
>12 yrs. of schooling	0.35	0.46	0.53	0.56

[a]Nonfinancial income is the sum of head's and wife's real labor income from all sources, and their combined transfer income expressed in 1982–1984 dollars. The income measure excludes head's and wife's Social Security income. Standard deviations are given in parentheses.

Table 12. Test of the null hypothesis of zero autocovariance in all time periods.[a]

Order	Test Stat.	d.f.	p-Value
1	506.81	28	0.00
2	57.88	27	0.00
3	45.35	26	0.01
4	23.65	25	0.54
5	23.76	24	0.48
≥3	465.96	351	0.00
≥4	425.18	325	0.00

[a]The test statistic is distributed as χ^2 with degrees of freedom equal to the number of (zero) restrictions (the number of unique autocovariances of a given order in the estimated variance–covariance matrix).

In column 2, I allow for a random walk and a deterministic growth-rate component in earnings. Monte Carlo results indicated that if both these components are present, the process should be empirically identified in small samples. In column 2, the estimate for the variance of the individual-specific growth rates binds at zero, while the estimate of the variance of the shock to the random-walk component equals 0.015 and is significant at the 1% level. An autoregressive parameter of the transitory process is estimated at about 0.37, capturing the fast decline of the empirical autocovariance function of labor income growth rates beyond the first order. The variance of the shocks to the transitory component, which also comprises the contribution of measurement error, is estimated at about 0.03.

nent since its autocovariance function encompasses the autocovariance function of both AR(1) and MA(1) transitory processes. The main result—that the variance of permanent shocks is significant and the estimated variance of the growth-rate heterogeneity is zero—holds for models with transitory components modeled as MA(1) or AR(1) processes.

TABLE 13. Estimates of income processes: PSID data.[a]

	(1) HIP	(2) Add RW	(3) Est. Pers.	(4) Same as col. 3 Set $\sigma^2_\beta = 0$	(5) Chang. Perm./ Trans. Var.	(6) Use Only First 10 acfs
$\hat{\sigma}^2_\beta$	0.0004	0.00	0.00	0.00	0.00	0.00
	(0.00004)	(0.00006)	(0.001)	—	—	(0.0002)
$\hat{\sigma}^2_\xi$	0.00	0.015	0.016	0.016	0.017	0.015
	—	(0.002)	(0.002)	(0.002)	(0.005)	(0.003)
$\hat{\phi}$	0.712	0.367	0.343	0.343	0.357	0.369
	(0.029)	(0.115)	(0.194)	(0.124)	(0.114)	(0.138)
$\hat{\theta}$	−0.187	−0.091	−0.081	−0.081	−0.105	−0.092
	(0.024)	(0.08)	(0.113)	(0.087)	(0.086)	(0.088)
$\hat{\sigma}^2_\varepsilon$	0.046	0.028	0.027	0.027	0.027	0.028
	(0.001)	(0.002)	(0.005)	(0.002)	(0.005)	(0.003)
$\hat{\phi}_{rw}$	0.0	1.0	0.992	0.992	1.0	1.0
	—	—	(0.158)	(0.009)	—	—
χ^2	793.32	697.05	694.38	696.96	492.25	636.89
(d.f.)	(431)	(430)	(429)	(430)	(376)	(430)

[a]The estimated income process is $y_{it} = \alpha_i + \beta_i h_{it} + p_{it} + \frac{1+\theta L}{1-\phi L}\varepsilon_{it} + u_{it,me}$, where $p_{it+1} = \phi_{rw} p_{it} + \xi_{it+1}$ and ϕ_{rw} denotes the autoregressive coefficient of a more persistent autoregressive process. Models are estimated by the equally weighted minimum distance method. The sample consists of 1916 male household heads with at least eight consecutive observations on labor income growth. Households from the Survey of Economic Opportunity (SEO) subsample are excluded. Standard errors are given in parentheses.

In column 3, I do not restrict the autoregressive parameter of a more persistent process to equal 1. The results are largely similar to those in column 2, but less precise. The estimated variance of growth-rate heterogeneity is still zero, while the autoregressive parameter of a more persistent process is close to 1.[26] Column 4 reports the results of the same model when the variance of growth-rate heterogeneity is set to zero. The results are quantitatively similar to those in column 3, while the parameters are estimated more precisely.

In column 5, I reestimate the model of column 4, allowing for time-varying permanent and transitory variances. I report the time averages of the estimated variances and the time averages of their standard errors. The results are similar to those in columns 2–4. The full sets of transitory and permanent variances, along with their standard errors, are presented in Table 14.

The off-diagonal elements of the empirical autocovariance matrix contain important information for identification of the variance of the growth-rate heterogeneity.

[26]The results of Tables 9 and 10 indicate that the estimated persistence of the permanent component is downward-biased in an unrestricted estimation, while the variance of growth-rate heterogeneity is somewhat upward- (downward-) biased when the permanent component is a random walk (a relatively less persistent autoregressive process). Since the estimated persistence in column 3 of Table 13 is very close to 1 and the estimated variance of growth-rate heterogeneity binds at 0, it is, perhaps, reassuring that the income process in my sample is best modeled as the sum of a random walk, a transitory component with low persistence, and no growth-rate heterogeneity.

TABLE 14. The variances of permanent and transitory shocks by year.[a]

Year	Trans. Shock	St. Err.	Perm. Shock	St. Err.
1969	0.01989	0.00428	0.01274[b]	—
1970	0.01461	0.00360	0.01274[b]	0.00581
1971	0.02635	0.00535	0.01299	0.00348
1972	0.02337	0.00609	0.01336	0.00461
1973	0.02541	0.00536	0.02177	0.00540
1974	0.02373	0.00467	0.01961	0.00452
1975	0.03137	0.00549	0.01223	0.00425
1976	0.03649	0.00599	0.02345	0.00592
1977	0.02708	0.00466	0.01329	0.00466
1978	0.02843	0.00466	0.00647	0.00350
1979	0.02586	0.00502	0.01467	0.00476
1980	0.02350	0.00503	0.01672	0.00470
1981	0.01538	0.00545	0.02725	0.00655
1982	0.02959	0.00526	0.02854	0.00791
1983	0.02309	0.00430	0.02605	0.00463
1984	0.02623	0.00507	0.02821	0.00575
1985	0.03125	0.00516	0.02216	0.00465
1986	0.02418	0.00401	0.01732	0.00501
1987	0.02679	0.00453	0.02440	0.00487
1988	0.02768	0.00470	0.01464	0.00482
1989	0.02421	0.00489	0.02935	0.00547
1990	0.02413	0.00433	0.01979	0.00486
1991	0.02039	0.00407	0.01659	0.00432
1992	0.02066	0.00346	0.02261	0.00482
1993	0.04993	0.00662	0.00766	0.00499
1994	0.03701	0.00600	0.00499	0.00404
1995	0.03103	0.00508	0.00618	0.00468
1996	0.02568	0.00464	0.00859[b]	0.00496
1997	0.03090	0.00699	0.00859[b]	—

[a]Estimates are from the model in Table 13, column 5.
[b]Variances of permanent shocks are restricted in estimation to be equal in these years.

For example, if the true income process contains a moving average process of order 1, higher-order autocovariances will be informative for identification of the variance of the growth-rate heterogeneity. The number of heads contributing toward the empirical autocovariance $\hat{\gamma}_k$ is, in general, smaller the larger is the lag length k, which separates the head's income observation at time t from the income observation at time $t+k$. Placing an equal weight on all the variances and autocovariances in estimation may bias an estimate of the growth-rate heterogeneity toward zero if higher-order empirical autocovariances are very close to zero and imprecisely estimated as, indeed, is found in empirical data. To take care of this concern, following Guvenen (2009), I reestimate the model utilizing only the first 10 empirical autocovariances and all the variances in estimation—column 6 Table 13. The main result remains unchanged: the growth-rate heterogeneity is estimated at zero, while the variance of permanent shocks is precisely estimated at about 0.015.

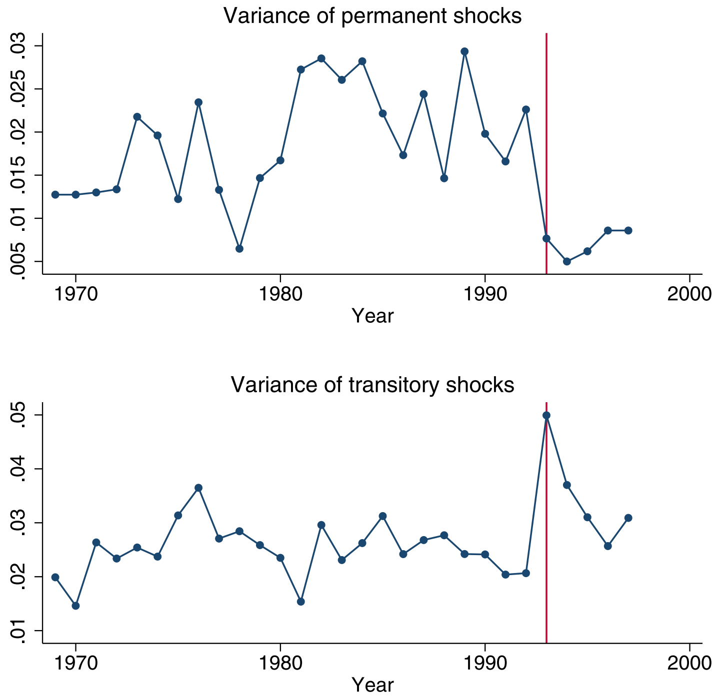

FIGURE 2. The variance of shocks to labor income by year. The variances of permanent and transitory shocks are estimated on the main sample fitting the model in Table 13, column 5. The vertical line is drawn for the survey year 1993, when the PSID switched to the electronic data collection method.

In Figure 2, I plot the resulting time series of the estimated variances of permanent and transitory shocks. It appears that an increase in the variance of heads' incomes in the early 1980s depicted in Figure 1 was largely due to the increase in the variance of permanent shocks during that period. The pattern of the variances of permanent shocks in the 1980s resembles that in Meghir and Pistaferri (2004), for their pooled sample that includes heads of household from the SEO subsample. It is also qualitatively similar to the hump-shaped pattern of the permanent volatility of *household* incomes in the 1980s reported in Blundell, Pistaferri, and Preston (2008).[27]

For robustness, I estimate the income process using separate samples for heads who dropped out from or just finished high school (columns 1 and 2 in Table 15) and those who finished some college, graduated from college or have education levels beyond college degree (columns 3 and 4 in Table 15). This roughly 50–50 split of the main sample allows precise estimates of the model parameters.

When the random-walk component is ignored in estimation, the variance of the deterministic growth-rate heterogeneity is substantial and statistically significant (columns 1 and 3 in Table 15); the estimated AR(1) persistence of the stochastic compo-

[27]In 1993, the PSID switched to the electronic data collection method. Presumably, the spike of the variance of transitory shocks in 1993—see the vertical line in Figure 2—is due to this change in the data collection method. The results are qualitatively similar if I do not use the data after 1992.

Table 15. Estimates of income processes for PSID data with the sample split by education.[a]

	High School Grad. or Less		Some College or More	
	(1)	(2)	(3)	(4)
$\hat{\sigma}^2_\beta$	0.0003	0.00	0.0004	0.00
	(0.00006)	(0.00008)	(0.00007)	(0.0001)
$\hat{\sigma}^2_\xi$	0.00	0.012	0.00	0.02
	—	(0.002)	—	(0.003)
$\hat{\phi}$	0.588	0.335	0.848	0.385
	(0.050)	(0.141)	(0.029)	(0.209)
$\hat{\theta}$	−0.165	−0.073	−0.179	−0.084
	(0.041)	(0.099)	(0.025)	(0.143)
$\hat{\sigma}^2_\varepsilon$	0.048	0.035	0.044	0.020
	(0.002)	(0.003)	(0.002)	(0.003)

[a]The estimated income process is $y_{it} = \alpha_i + \beta_i h_{it} + p_{it} + \frac{1+\theta L}{1-\phi L}\varepsilon_{it} + u_{it,me}$, where $p_{it+1} = p_{it} + \xi_{it+1}$. Models are estimated by the equally weighted minimum distance method. In columns 1 and 2, the sample consists of 1011 male household heads with at least eight consecutive observations on labor income growth whose education levels do not exceed high school. In columns 3 and 4, the sample consists of 905 male household heads with at least eight consecutive observations on labor income growth who finished some college or graduated from college. Households from the Survey of Economic Opportunity (SEO) subsample are excluded. Standard errors are given in parentheses.

nent is moderate, ranging from about 0.85 for the college sample to 0.59 for high school graduates/dropouts. Similar to the main results, in the model that includes both the growth-rate heterogeneity and the permanent random-walk component, the estimated variance of the growth-rate heterogeneity equals zero (columns 2 and 4). Interestingly, the estimated variance of permanent shocks is higher for more educated heads, while the estimated variance of the shocks to the transitory component is higher for less educated heads.

In this paper I focused on the moments in growth rates. Recently, Heathcote, Perri, and Violante (2010) estimated the income process composed of a random walk and a purely transitory shock, and warned of the inconsistency of the estimates that results from using the moments of log income in levels or differences. In particular, they found that the variance of permanent (transitory) shocks is relatively larger when one uses the moments in differences (levels). To my knowledge, this issue has not been properly addressed in the literature yet. The inconsistency of the estimates in levels and differences can be, for example, due to rare events and/or shocks such as job mobility that affect the moments in differences and levels differently. Low, Meghir, and Pistaferri (2010), using a model of consumption and employment choices, showed the importance of these rare events for the estimated size of permanent shocks when using the moments in growth rates. I view this issue as separate from my paper and one that deserves future research.

Discussion in Section 2.2.1 suggested that the estimated variance of the growth-rate heterogeneity should be inversely related to the time dimension of the sample size if the true process contains a random-walk component and the income process is estimated as HIP. To support this result, I simulated the model of column 2 in Table 13 for samples with different time dimension but the same group of individuals.

Table 16. The time span of a sample and estimated growth-rate heterogeneity: simulated data.[a]

Time Span, T	(1) True: HIP $\hat{\sigma}_\beta^2$	(2) True: RIP with R.W. $\hat{\sigma}_\beta^2$
10	0.00048	0.001
	(0.0002)	(0.0002)
15	0.00045	0.0008
	(0.00009)	(0.0001)
20	0.00044	0.0006
	(0.00006)	(0.00007)
25	0.00043	0.0005
	(0.00005)	(0.00007)
30	0.00043	0.00045
	(0.00006)	(0.00006)

[a] In both columns, the estimated income process is $y_{it} = \alpha_i + \beta_i h_{it} + \frac{1+\theta L}{1-\phi L}\varepsilon_{it}$. In column 1, the true process is the same as the estimated process. In simulations, the parameters are taken from column 1 of Table 13. In column 2, the true process is $y_{it} = \alpha_i + p_{it} + \frac{1+\theta L}{1-\phi L}\varepsilon_{it}$, where $p_{it} = p_{it-1} + \xi_{it}$. In simulations, the parameters are taken from column 2 of Table 13. Models are estimated by the equally weighted minimum distance method. Standard errors are given in parentheses and are calculated as the standard deviations of the estimated growth-rate heterogeneity across 100 model simulations.

The estimated (misspecified) HIP model always returns nonzero and significant estimates of the growth-rate heterogeneity—column 2 of Table 16—which are higher for samples with a smaller time dimension. I also performed simulations of the model in column 1 of Table 13. The results are in column 1 of Table 16. If the true model is HIP, the variance of growth-rate heterogeneity should not depend on the time dimension of the sample size, as the results in column 1 suggest. If, for the same cross section of heads, an estimate of the growth-rate heterogeneity is found to be systematically different for different time dimensions of the sample, this will present some additional evidence against the hypothesis that heads' idiosyncratic income growth rates systematically and deterministically differ over the life cycle.

Figure 3 graphically presents just outlined arguments using PSID data. First, I select a sample of 1157 PSID heads who have at least five consecutive income observations during 1968–1977 and estimate the HIP process for idiosyncratic incomes for that sample. This gives me the first point in the graph in Figure 3, panel (a). I then extend the time dimension of the initial sample to 1978, keeping the cross-sectional dimension fixed, and estimate the HIP process for that sample. I continue this procedure until I arrive at the sample that spans the period 1969–1997 for those 1157 heads, the longest possible period.[28] The results in Figure 3 are quite telling: it appears that the estimated growth-rate heterogeneity is larger for smaller time dimensions of the sample size, even though the samples contain the same group of individuals and one would expect the estimated growth-rate heterogeneity to be the same. Next, on those samples, I perform a series

[28] The Monte Carlo results just described replicated these samples in terms of the number of individuals, the number of person-year observations, and the cross-sectional distribution of age by year.

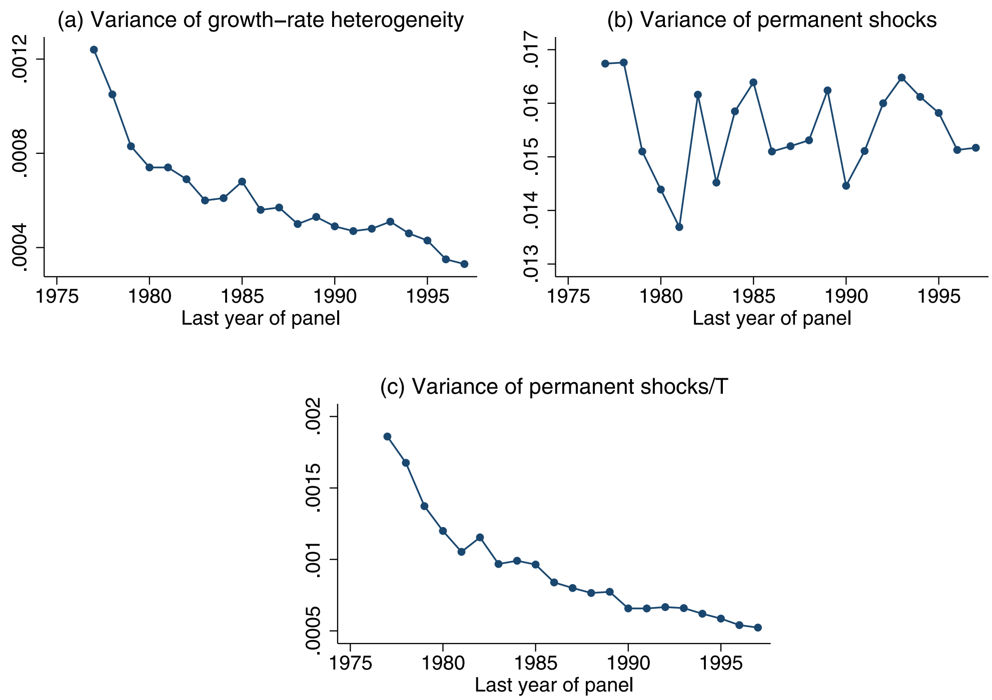

FIGURE 3. The variance of growth-rate heterogeneity and the time dimension of the sample, T. In panel (a), the first point on the graph is an estimate of the variance of growth-rate heterogeneity for a sample of 1157 PSID heads with at least five consecutive income observations during 1968–1977; all subsequent points are the estimates of the variance of growth-rate heterogeneity using samples that contain the same heads and income information for 1968–1978, 1968–1979, all the way up to 1968–1997. In panel (b), the same samples are used to estimate the income process that includes a permanent random-walk component and an ARMA(1, 1) transitory process. The estimates of the variance of permanent shocks are then divided by the time dimension of the sample size to produce the graph in panel (c).

of estimations, assuming that the true process contains a random walk and the transitory process is modeled as an ARMA(1, 1) process. The results are plotted in panel (b) of Figure 3. One could think that those are the unconditional estimates of the permanent variance for different time dimensions: the average permanent variance will increase if the marginal variance is higher than the average variance and vice versa. In panel (c) of Figure 3, I divide the estimated permanent variances in the rightmost panel of Figure 3 by the time dimension of the sample utilized for their estimation. Remarkably, the series of the estimated growth-rate heterogeneity in the leftmost graph is quite similar to the series of the variance of permanent shocks scaled by the inverse of the time dimension of the sample, as one would expect if the true process contains a random-walk component and no growth-rate heterogeneity.

There is some evidence, which does not rely on estimation of income processes, interpreted by some researchers as favoring income models with heterogeneous income profiles. Haider and Solon (2006) and Böhlmark and Lindquist (2006) studied the association between current and lifetime income over the life cycle for U.S. and Swedish samples, respectively. Specifically, they focused on the life-cycle variation in the slope coefficient from the regression $y_{ia} = \beta_a V_i + \varepsilon_{ia}$, where y_{ia} is individual i's log income at age a, V_i is individual i's log lifetime income, calculated as (the log of) the annuity value of the discounted sum of annual real incomes observed for individual i, and ε_{ia} is individual i's regression error at age a. Haider and Solon (2006) found that β_a is estimated at about 0.20 at age 19, steadily increases afterward, equals 1 at age 34, and levels off for the rest of the life cycle. Böhlmark and Lindquist (2006), for a much larger Swedish sample, found that $\hat{\beta}_a$ starts at about 0.20 at age 19, crosses 1 at age 34, and peaks at 1.45 at age 48. The latter authors interpreted this result as evidence favoring the presence of heterogeneous income profiles—income is low in the beginning of the life cycle and is well below the lifetime income (which is estimated to be time-invariant by the authors); income then steadily grows until it exceeds the lifetime income in the later part of the life cycle. This result is, however, also true for income processes that contain random walks and do not have deterministic idiosyncratic trends. Using the estimates of the RIP process in this paper, I was able to replicate, in simulations not reported here, the pattern of $\hat{\beta}_a$'s found in Böhlmark and Lindquist (2006). The intuition behind this result is the following. Note that $\hat{\beta}_a = \frac{\text{cov}(y_{ia}, V_i)}{\text{var}(V_i)}$. While the denominator is constant over the life cycle, the cross-sectional covariance between current incomes and lifetime incomes will be growing over the life cycle, since current incomes will accumulate random-walk shocks over the life cycle and will, therefore, co-vary more strongly with lifetime incomes, which aggregate all the permanent shocks to individual incomes over the entire life cycle.

Summarizing, for the samples utilized in this study, it appears that I can reject the HIP model. The RIP model with a permanent random-walk component and a transitory mean-reverting component cannot be rejected.

4. Conclusion

I estimate idiosyncratic labor income processes on simulated and empirical data. The main results of a Monte Carlo study using unbalanced panel data are the following. It is possible to identify a general process containing all the elements of the HIP and the RIP models. The most important elements are the growth-rate heterogeneity and the variance of a random-walk component. For simulated data in first differences, I show that both these elements, if present, should be recovered precisely in empirical estimations utilizing small unbalanced samples. The results on simulated data confirm another important finding of this paper: if the true income process is the sum of a random walk and persistent components (i.e., the RIP) and the random walk is ignored in estimation, the misspecified HIP model recovers significant and substantial growth-rate heterogeneity and modest persistence.

I use data for male household heads from the 1968–1997 waves of the PSID to estimate idiosyncratic labor income processes. I find that the estimated variance of the deterministic growth-rate heterogeneity is zero, while the estimated variance of the permanent component is significant and substantial.

The results of this paper are important for understanding a number of issues. Among them is the choice of an appropriate model of the heterogeneity in individual and household idiosyncratic incomes used in macromodels. The process, best fitting the data utilized in this paper, also places restrictions on the models that make earnings an endogenous variable.

In this paper, I use only income data to identify the variances of idiosyncratic permanent and transitory shocks. Perhaps more accurate estimates of the variances could be obtained by jointly studying household choices and income data. For recent attempts at this approach, see Blundell and Preston (1998), Hryshko (2007), and Blundell, Pistaferri, and Preston (2008) (in the context of RIP), and Guvenen and Smith (2008) (in the context of HIP), who utilized data on household income and consumption choices.

Appendix: Estimation details

In my Monte Carlo simulations, I am assuming that $\tau_{it} = 0$ and $p_{it} = 0$ if $h_{it} = 0$, that is, a head with no labor market experience entering the labor market at time $t + 1$ is "endowed" with zero permanent and transitory components of earnings.

The theoretical autocovariance moments for a model with the transitory component modeled as an AR(1) process are shown in equations (10)–(12). If the transitory component is a moving average process of order 1, see the autocovariance function in the text in equations (5)–(8). The empirical moments, taking into account that the data used in estimations are unbalanced, are calculated as

$$\operatorname{vech}\left(\sum_{i=1}^{N} \tilde{y}_i \tilde{y}_i'\right) \Big/ N_{tt'},$$

where $\tilde{y}_i = (\Delta y_{i2}, \Delta y_{i3}, \ldots, \Delta y_{iT})$; N is the total number of heads in the sample; $N_{tt'}$ is a vector with row dimension $\frac{T(T+1)}{2}$; N_{11} is the number of heads contributing toward estimation of the variance in period 1 ($t = 1$, $t' = 1$); N_{12} is the number of heads contributing toward estimation of the first-order autocovariance between periods 1 and 2 ($t = 1$, $t' = 2$), and so forth. Note that if the head's income is missing, say, in period 1, this head's contributions toward the variance at time 1 and all the sample autocovariances involving this period are zero. The vector of data moments used in estimation is $m^d = \operatorname{vech}(\sum_{i=1}^{N} \tilde{y}_i \tilde{y}_i')/N_{tt'}$; the row dimension of m^d is $\frac{T(T+1)}{2}$. The model parameters, Θ, are recovered by minimizing a squared distance function $[m(\Theta) - m^d]' I [m(\Theta) - m^d]$, where I is an identity matrix with the row dimension $\frac{T(T+1)}{2}$.

Standard errors of the parameters are calculated as the square roots of the diagonal of $(G_\Theta' G_\Theta)^{-1} G_\Theta' V G_\Theta (G_\Theta' G_\Theta)^{-1'}$, where $G_\Theta = \frac{\partial}{\partial \Theta}[m(\hat{\Theta}) - m^d]$, a vector with the row dimension $\frac{T(T+1)}{2}$, and the column dimension equal to the row dimension of the vector of estimated parameters; V is equal to $\sum_{i=1}^{N} (m_i - m^d)(m_i - m^d)'/N_V$, where $m_i = \operatorname{vech}(\tilde{y}_i \tilde{y}_i')$, and the klth element of N_V is calculated as $N_V^{kl} = N_{tt'}^k N_{tt'}^l$, where $N_{tt'}^k$ is the kth element of $N_{tt'}$.

References

Abowd, J. and D. Card (1989), "On the covariance structure of earnings and hours changes." *Econometrica*, 57 (2), 411–445. [181, 182, 184]

Altonji, J. G. and L. M. Segal (1996), "Small-sample bias in GMM estimation of covariance structures." *Journal of Business & Economic Statistics*, 14 (3), 353–366. [183]

Baker, M. (1997), "Growth-rate heterogeneity and the covariance structure of life-cycle earnings." *Journal of Labor Economics*, 15 (2), 338–375. [181, 182, 183, 191, 192]

Blundell, R., L. Pistaferri, and I. Preston (2008), "Consumption inequality and partial insurance." *American Economic Review*, 98 (5), 1887–1921. [201, 206]

Blundell, R. and I. Preston (1998), "Consumption inequality and income uncertainty." *Quarterly Journal of Economics*, 113, 603–640. [206]

Böhlmark, A. and M. J. Lindquist (2006), "Life-cycle variations in the association between current and lifetime income: Replication and extension for Sweden." *Journal of Labor Economics*, 24 (4), 879–896. [204, 205]

Bound, J., C. Brown, and N. Mathiowetz (2001), "Measurement error in survey data." In *Handbook of Econometrics*, Vol. 5 (J. J. Heckman and E. Leamer, eds.), Chap. 59, 3705–3843, North-Holland, New York. [188]

Carroll, C. D. (1992), "The buffer-stock theory of saving: Some macroeconomic evidence." *Brookings Papers on Economic Activity*, 23, 61–156. [181]

Carroll, C. D. and A. A. Samwick (1997), "The nature of precautionary wealth." *Journal of Monetary Economics*, 40 (1), 41–71. [179, 181]

Castañeda, A., J. Díaz-Giménez, and J.-V. Ríos-Rull (2003), "Accounting for the U.S. earnings and wealth inequality." *Journal of Political Economy*, 111 (4), 818–857. [179]

De Santis, M. (2007), "Individual consumption risk and the welfare cost of business cycles." *American Economic Review*, 97 (4), 1488–1506. [179]

Fitzgerald, J., P. Gottschalk, and R. Moffitt (1998), "An analysis of sample attrition in panel data: The Michigan panel study of income dynamics." *Journal of Human Resources*, 33, 251–299. [182]

Friedman, M. (1957), *A Theory of the Consumption Function*. Princeton University Press, Princeton, NJ. [181]

Friedman, M. and S. Kuznets (1954), *Income From Independent Professional Practice*. National Bureau of Economic Research, New York. [181]

Galbraith, J. and V. Zinde-Walsh (1994), "A simple noniterative estimator for moving average models." *Biometrika*, 81, 143–155. [182]

Guvenen, F. (2007), "Learning your earning: Are labor income shocks really very persistent?" *American Economic Review*, 97 (3), 687–712. [178, 179]

Guvenen, F. (2009), "An empirical investigation of labor income processes." *Review of Economic Dynamics*, 12 (1), 58–79. [178, 180, 181, 182, 183, 188, 189, 191, 192, 200]

Guvenen, F. and A. Smith (2008), "Inferring labor income risk from economic choices: An indirect inference approach." Mimeo, University of Minnesota. [180, 206]

Haider, S. and G. Solon (2006), "Life-cycle variation in the association between current and lifetime earnings." *American Economic Review*, 96 (4), 1308–1320. [204, 205]

Haider, S. J. (2001), "Earnings instability and earnings inequality of males in the United States: 1967–1991." *Journal of Labor Economics*, 19 (4), 799–836. [182]

Hamilton, J. (1994), *Time Series Analysis*. Princeton University Press, Princeton, NJ. [185]

Hause, J. C. (1980), "The fine structure of earnings and the on-the-job training hypothesis." *Econometrica*, 48 (4), 1013–1029. [180, 181]

Heathcote, J., F. Perri, and G. L. Violante (2010), "Unequal we stand: An empirical analysis of economic inequality in the United States: 1967–2006." *Review of Economic Dynamics*, 13 (1), 15–51. [202]

Hryshko, D. (2007), "Excess smoothness of consumption in an estimated life cycle model." Mimeo, University of Alberta. [206]

Krebs, T. (2003), "Human capital risk and economic growth." *Quarterly Journal of Economics*, 118 (2), 709–744. [179]

Lillard, L. A. and C. W. A. Panis (1998), "Panel attrition from the panel study of income dynamics: Household income, marital status, and mortality." *Journal of Human Resources*, 33, 437–457. [182]

Lillard, L. A. and Y. Weiss (1979), "Components of variation in panel earnings data: American scientists 1960–1970." *Econometrica*, 47 (2), 437–454. [180, 181, 182]

Low, H., C. Meghir, and L. Pistaferri (2010), "Wage risk and employment risk over the life cycle." *American Economic Review*, 100, 1432–1467. [202]

MaCurdy, T. E. (1982), "The use of time series processes to model the error structure of earnings in a longitudinal data analysis." *Journal of Econometrics*, 18 (1), 83–114. [181, 183, 189]

Meghir, C. and L. Pistaferri (2004), "Income variance dynamics and heterogeneity." *Econometrica*, 72 (1), 1–32. [179, 181, 182, 183, 184, 188, 195, 196, 197, 201]

Moffitt, R. A. and P. Gottschalk (1995), "Trends in the covariance structure of earnings in the U.S.: 1969–1987." Mimeo, Johns Hopkins University. [179]

Muth, J. (1960), "Optimal properties of exponentially weighted forecasts." *Journal of the American Statistical Association*, 35, 299–306. [181]

Newey, W. K. (1985), "Generalized method of moments specification testing." *Journal of Econometrics*, 29, 229–256. [187, 189, 191, 192, 193, 194, 195, 196]

Scholz, J. C., A. Seshadri, and S. Khitatrakun (2006), "Are Americans saving "optimally" for retirement?" *Journal of Political Economy*, 114 (4), 607–643. [179]

Submitted September, 2010. Final version accepted August, 2011.