Supplement to "Rising indebtedness and temptation: A welfare analysis": Appendices

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Makoto Nakajima
Research Department, Federal Reserve Bank of Philadelphia

Appendix A: Additional details of calibration

Figure A.1 shows the age-specific survival probability, which is used as $\{s_i\}_{i=1}^{I}$ in the calibration. Figure A.2 compares the discount factors of the standard exponential discounting and hyperbolic discounting models for periods (years) 1–50. The calibrated β and δ are used. The figure shows that the discount factor drops substantially more from period 1 to period 2 in the case of the hyperbolic discounting preferences. On the other hand, the discount factor applied to utility in the distant future is higher for the hyperbolic discounting model. Laibson (1997) argued that housing, from which inhabitants can enjoy utility as long as they own it and live in it, has an extra value for hyperbolic discounting consumers, since the dividends can be enjoyed for a long period of time. Figure A.3 shows the average life-cycle profile of labor productivity. This is used as $\{e_i\}_{i=1}^{I}$ in the calibration.

Appendix B: Computation algorithm

I first describe the computational algorithm to solve the steady-state equilibrium of the model with temptation. Since the focus is the steady-state equilibrium, I drop the time script in the algorithm. The solution method for the model without temptation (i.e., exponential discounting model) is straightforward and thus is omitted. Adding the labor–leisure decision is also straightforward.

Algorithm 1 (Computation Algorithm for Solving Steady-State Equilibrium).

1. Set the initial guess of the aggregate capital stock K^0 and the per-consumer transfer d^0. Notice that the aggregate labor supply $\overline{L}$ can be computed independently from the model since there is no labor supply decision.

Makoto Nakajima: makoto.nakajima@phil.frb.org
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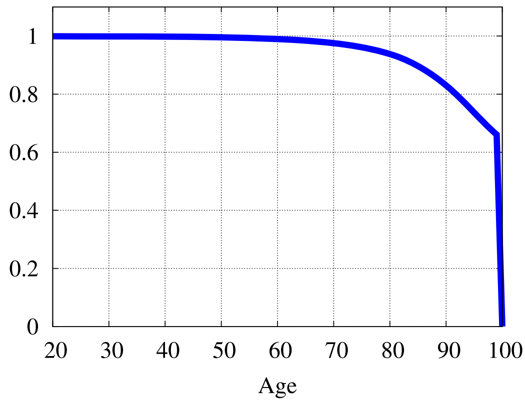

FIGURE A.1. Age-specific survival probability.

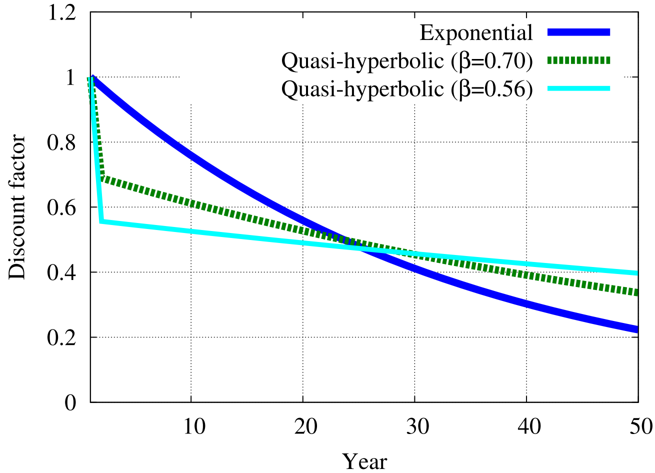

FIGURE A.2. Comparison of discount factors.

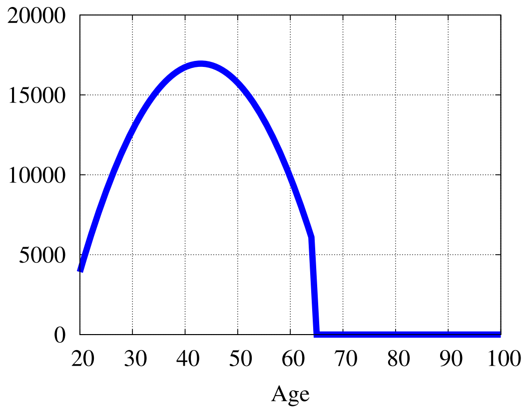

FIGURE A.3. Average life-cycle profile of labor productivity.

2. Given K^0 and $\overline{L}$, compute the interest rate r and the wage w. The transfer used in the iteration is equal to the guess, that is, $d = d^0$. The Social Security benefits $\overline{b}$ can be computed using the government budget constraint (9) together with w and the type distribution of consumers with respect to age and labor productivity. Once $\overline{b}$ is obtained, $\{b_i\}_{i=1}^{I}$ is set as $b_i = \overline{b}$ for $i \geq I_R$ and $b_i = 0$ for $i < I_R$.

3. Given $\{r, w, d, \{b_i\}_{i=1}^{I}\}$, solve the consumer's optimization problem using backward induction.

(a) Set $V(I+1, p, a) = 0$ for all p and a.

(b) Solve the problem of the age-I consumer, using the Bellman equation (1) for all (p, a). The optimal level a' is obtained by basically comparing values conditional on a' and choosing a' associated with the highest value. Notice that since $\gamma \to \infty$, the optimal a' is chosen to maximize the temptation utility, while the value function is updated using the self-control utility.

(c) With $V(I, p, a)$ at hand, we can solve the problem of the age-$I-1$ consumer. Keep going back in the same way until the value function and the optimal decision rule for the age-1 (initial age) consumer are obtained.

4. Using the obtained optimal decision rule $g^a(i, p, a)$, simulate the model.

(a) Set the type distribution for the newborns ($i = 1$), which is exogenously given. In particular, all newborns have $i = 1$ and $a = 0$. Initial p is distributed according to π_p^0.

(b) Update the type distribution using the stochastic process for p and the optimal decision rule $g^a(i, p, a)$, and obtain the type distribution for $i = 2$.

(c) Keep updating until $i = I$ (last age).

5. Compute the aggregate capital stock K^1 and the total amount of the accidental bequests implied by the simulated distribution. Notice that consumers survive according to the age-dependent survival probability and that there is population growth, which makes the size of the younger population larger. Make these adjustments when computing the aggregate capital stock and the total amount of the accidental bequests. Specifically, when the measure of age-1 consumers is normalized to 1, the measure of age-i consumers, $\tilde{\mu}_i$, can be represented as

$$\tilde{\mu}_i = \frac{1}{(1+\nu)^{i-1}} \prod_{j=0}^{i-1} s_j,$$

where $s_0 = 1$. Once the aggregate amount of the accidental bequests is computed, we can compute the per-consumer lump-sum transfer d^1 using the government budget constraint (10).

6. Compare $\{K^0, d^0\}$ and $\{K^1, d^1\}$. If they are closer than the predetermined tolerance level, stop. Otherwise, update $\{K^0, d^0\}$ and go back to step 2.

Next, I describe the solution algorithm of an equilibrium that features the deterministic transition between two steady states. The first step is to obtain the two steady states using Algorithm 1. Denote the initial and the new steady state by $t = 0$ and $t = \infty$, respectively. Set the initial distribution along the transition path μ_0 as the type distribution of consumers in the initial steady state, and denote the value at the end of the transition $V_\infty(i, p, a)$ as the value function in the new steady state. The only difference between the two steady states is the borrowing limit $\underline{a}$; total factor productivity Z is assumed to be constant over time. I also assume that the transition is complete after $T < \infty$ periods. Since the model economy converges to the new steady state only asymptotically, a large T is desirable for a good approximation. Now, in period 0, the economy is in the initial steady state, but in period 1, the transition, in particular the sequence of the borrowing limit $\{\underline{a}_t\}_{t=1}^T$, is revealed to consumers. Let $\underline{a}_1 = \underline{a}_0 = 0$, let $\underline{a}_t = \underline{a}_\infty$ (the borrowing limit in the 2000 economy) for $t = \tilde{T}, \tilde{T}+1, \tilde{T}+2, \ldots, T$, and let $\underline{a}_t$ gradually increase between period 1 and period $\tilde{T} < T$. I set $\tilde{T} = 30$. Since $t = 0$ corresponds to 1970 (initial steady state without borrowing) and one period is a year, then $t = \tilde{T} = 30$ corresponds to 2000. After 2000, the borrowing limit is assumed to remain at the same level as in 2000.

ALGORITHM 2 (Computation Algorithm for Solving Equilibrium Transition Path).

1. Set the initial guess of the sequence $\{K_t^0, d_t^0\}_{t=0}^T$. Notice that the sequence of aggregate labor supply $\{\overline{L}_t\}_{t=0}^T$ can be computed independently from the model.

2. Given $\{K_t^0, d_t^0, \overline{L}_t\}_{t=0}^T$, compute the sequence $\{r_t, w_t, d_t, \{b_{t,i}\}_{i=1}^I\}_{t=0}^T$.

3. Given $\{r_t, w_t, d_t, \{b_{t,i}\}_{i=1}^I\}_{t=0}^T$ and $\{\underline{a}_t\}_{t=0}^T$, solve the consumer's optimization problem using backward induction.

(a) Start from period T. Notice that we know the value function $V_{T+1}(i, p, a) = V_\infty(i, p, a)$ for all (i, p, a), since the economy is assumed to have converged to the new steady state in period T.

(b) Solve the consumer's problem for all (i, p, a) in period T, given $V_{T+1}(i, p, a)$. The solution method for the model with temptation is the same as in the steady-state equilibrium described in Algorithm 1. The optimal decision rule in period T, $g_T^a(i, p, a)$, and the value function for period T, $V_T(i, p, a)$, are obtained. Notice that since the value function for period $T+1$ is known, there is no need to go back from age I as in Algorithm 1.

(c) Keep going back until $t = 0$.

4. Using the obtained sequence of optimal decision rules $\{g_t^a(i, p, a)\}_{t=0}^T$, simulate the model.

(a) The type distribution in period 0 is given by μ_0.

(b) Update the type distribution using the stochastic process for p and the optimal decision rule for period t, $g_t^a(i, p, a)$, and obtain the type distribution in period 1 (μ_1). Make sure to normalize the population size each period.

(c) Keep updating until period T (last period).

5. Compute $\{K_t^1, d_t^1\}_{t=0}^T$ using the sequence of type distribution $\{\mu_t\}_{t=0}^T$ generated in the last step.

6. Compare $\{K_t^0, d_t^0\}_{t=0}^T$ and $\{K_t^1, d_t^1\}_{t=0}^T$. If they are closer than the predetermined tolerance level, stop. Otherwise, update $\{K_t^0, d_t^0\}_{t=0}^T$ and go back to step 2.

Reference

Laibson, D. (1997), "Golden eggs and hyperbolic discounting." *Quarterly Journal of Economics*, 112 (2), 443–477. [1]

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