Supplement to "Effects of school reform on education and labor market performance: Evidence from Chile's universal voucher market"

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Supplemental appendix

This appendix describes the method of estimating the dynamic schooling and work model presented in the main paper. The model is estimated by maximum likelihood. Let O_{it} represent the outcomes (education choices, work choices, observed wages) of individual i and age a. Also, let I_i denote the set of initial conditions for that individual (family background variables, type of primary school attended). The contribution to the likelihood of individual i is given by

$$L_i = \sum_{k=1}^{K} \Pr(O_{ia}, O_{ia-1}, \dots, O_{ia_0}; \mu_k = 1, I_i) \Pr(\mu_k = 1 | I_i),$$

where $\Pr(\mu_k = 1|I_i)$ denotes the type probability, which depends on initial conditions, which in our application represent family background, socioeconomic status, parental education levels, and numbers of siblings. The unobserved type is assumed to be known to the individual but not to the econometrician; the outside summation integrates over the type probabilities. The likelihood can be written as the product over the age-specific choice probabilities:

$$L_i = \sum_{k=1}^{K} \prod_{a=a_0}^{A} \Pr(O_{ia} | O_{ia-1}, \dots, O_{ia_0}; \mu_k = 1, I_i) \Pr(\mu_k = 1 | I_i).$$

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To illustrate the calculation of the likelihood, suppose that the jth alternative chosen by individual i is to work, so that we observe a wage at age a. The probability of observing that choice and wage outcome conditional on the state space (which includes $O_{ia-1}, \dots, O_{ia_0}$, I and type) is

$$\begin{aligned}\Pr(O_{ia}|O_{ia-1}, \dots, O_{ia_0}; \mu_k = 1, I_i) &= \Pr(d^j(a) = 1, w_a|\Omega(a), I, \mu_k = 1) \\ &= \Pr(d^j(a)|w_a, \Omega(a), I) f(w_a|\Omega(a), I, \mu_k = 1),\end{aligned}$$

where $f(w_a|\Omega(a), I, \mu_k = 1)$ is the wage density.

The overall likelihood for $i = 1, \dots, N$ individuals is the product over the individual likelihoods:

$$L = \prod_{i=1}^{N} L_i.$$

To complete the description of the model, we need to specify the functional form for the type probabilities: They assume that type depends on parents' education, number of siblings, and family socioeconomic status (the initial conditions, denoted I_i) in the manner

$$P(\text{type} = k|I_i) = \frac{\exp(I_i'\tau)}{1 + \exp(I_i'\tau)}.$$

To estimate the probabilities $\Pr(O_{it}|O_{it-1}, \dots, O_{it_0}; \mu_k = 1)$ in a way that improves the empirical performance of the estimator, we use the kernel smoothed frequency simulator proposed by McFadden (1989). For each set of error term draws, we calculate

$$\frac{\exp\left\{\dfrac{V^i(a) - \max(V^j(a))}{\tau}\right\}}{\displaystyle\prod_{l=1}^{J} \exp\left\{\dfrac{V^l(a) - \max(V^j(a))}{\tau}\right\}}$$

and the average over all draws is the estimator for the choice probability, conditional on wages. Here, $V^i(a)$ is the value function associated with the choice that person i made at age a, $\max(V^j(a))$ is the value function associated with the maximal choice, and τ is a smoothing parameter.

Reference

McFadden, D. (1989), "A method of simulated moments for estimation of discrete response models without numerical integration." *Econometrica*, 57, 995–1026. [2]