Supplement to “On the welfare cost of consumption fluctuations in the presence of memorable goods”

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Appendix A: Data

Income Categories: We define disposable income as income before tax minus reported federal, state, and local income taxes payments, property tax not reported elsewhere and other tax (net of tax refunds), deductions for social security and pension plans. Household income before tax includes wages and salaries, net business income, net farm income, rents income, dividend income, interest income, pension income, social security and railroad retirement income, supplemental security income, unemployment compensation, workers' compensation and veterans benefits, welfare received, scholarship, food stamps, contributions received from others with alimony/child support, meals received as pay, rent received as pay, and lump sum receipts and lump sum child support payment.

Table A.1. Average monthly income and consumption expenditures—Sample.

<table>
<thead>
<tr>
<th></th>
<th>Disp Income</th>
<th>Total Outlays</th>
<th>ND+Memorables</th>
<th>ND</th>
<th>Memorables</th>
<th>Durables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>1972.32</td>
<td>1484.61</td>
<td>1193.87</td>
<td>905.54</td>
<td>288.33</td>
<td>290.74</td>
</tr>
</tbody>
</table>

Table A.2. Average demographic characteristics—Sample.

<table>
<thead>
<tr>
<th></th>
<th>Age of Head</th>
<th>Male Head</th>
<th>White Head</th>
<th>Married</th>
<th>High School Above</th>
<th>Family Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>41.14</td>
<td>0.65</td>
<td>0.84</td>
<td>0.59</td>
<td>0.85</td>
<td>2.87</td>
</tr>
</tbody>
</table>

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### Table A.3. Underlying frequencies of CEX expenditure categories.

<table>
<thead>
<tr>
<th>Number</th>
<th>Categories</th>
<th>Frequency</th>
<th>Number</th>
<th>Categories</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Food at home</td>
<td>Quarterly</td>
<td>1</td>
<td>Food away from home excl. on trips</td>
<td>Quarterly</td>
</tr>
<tr>
<td>2</td>
<td>Food at school</td>
<td>Monthly</td>
<td>2</td>
<td>Food for catered affairs</td>
<td>Monthly</td>
</tr>
<tr>
<td>3</td>
<td>Meals received as pay</td>
<td>Quarterly</td>
<td>3</td>
<td>Food on trips/vacations</td>
<td>Monthly</td>
</tr>
<tr>
<td>4</td>
<td>Tobacco products</td>
<td>Quarterly</td>
<td>4</td>
<td>Alcohol out excl. on trips/vacations</td>
<td>Quarterly</td>
</tr>
<tr>
<td>5</td>
<td>Alcohol home</td>
<td>Quarterly</td>
<td>5</td>
<td>Alcohol on trips</td>
<td>Monthly</td>
</tr>
<tr>
<td>6</td>
<td>Personal care services</td>
<td>Quarterly</td>
<td>6</td>
<td>Entertainment fees and admissions</td>
<td>Monthly</td>
</tr>
<tr>
<td>7</td>
<td>Maintenance, insurance, etc.—Owned housing</td>
<td>Monthly</td>
<td>7</td>
<td>Entertainment fees and admissions on trips/vacations</td>
<td>Monthly</td>
</tr>
<tr>
<td>8</td>
<td>Tenant-occupied nonfarm dwelling—Rent</td>
<td>Monthly</td>
<td>8</td>
<td>Other entertainment services and rental</td>
<td>Monthly</td>
</tr>
<tr>
<td>9</td>
<td>Rental equivalence of owned home</td>
<td>Annually</td>
<td>9</td>
<td>Other entertainment services and rental on trips</td>
<td>Monthly</td>
</tr>
<tr>
<td>10</td>
<td>Rent received as pay</td>
<td>Quarterly</td>
<td>10</td>
<td>Lodging on trips/vacations</td>
<td>Monthly</td>
</tr>
<tr>
<td>11</td>
<td>Lodging at school</td>
<td>Monthly</td>
<td>11</td>
<td>Vehicle maintenance, rental etc. on trips/vacations</td>
<td>Monthly</td>
</tr>
<tr>
<td>12</td>
<td>Electricity</td>
<td>Monthly</td>
<td>12</td>
<td>Gasoline and oil on trips/vacations</td>
<td>Monthly</td>
</tr>
<tr>
<td>13</td>
<td>Gas</td>
<td>Monthly</td>
<td>13</td>
<td>Public transportation on trips/vacations excl. airline</td>
<td>Monthly</td>
</tr>
<tr>
<td>14</td>
<td>Water and other sanitary services</td>
<td>Monthly</td>
<td>14</td>
<td>Airline fares</td>
<td>Monthly</td>
</tr>
<tr>
<td>15</td>
<td>Fuel oil and coal</td>
<td>Monthly</td>
<td>15</td>
<td>Photographic services and rental</td>
<td>Monthly</td>
</tr>
<tr>
<td>16</td>
<td>Telephone service</td>
<td>Monthly</td>
<td>16</td>
<td>Charitable giving</td>
<td>Monthly</td>
</tr>
<tr>
<td>17</td>
<td>Domestic service</td>
<td>Monthly</td>
<td>17</td>
<td>Clothing and shoes</td>
<td>Monthly</td>
</tr>
<tr>
<td>18</td>
<td>Other household operation</td>
<td>Monthly</td>
<td>18</td>
<td>Jewelry and watches</td>
<td>Monthly</td>
</tr>
<tr>
<td>19</td>
<td>Vehicle registration</td>
<td>Monthly</td>
<td>19</td>
<td>Clothing services, including watch and jewelry repair</td>
<td>Monthly</td>
</tr>
<tr>
<td>20</td>
<td>Vehicle maintenance, rental etc.</td>
<td>Monthly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>Gasoline and oil</td>
<td>Monthly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number</td>
<td>Categories</td>
<td>Frequency</td>
<td>Number</td>
<td>Categories</td>
<td>Informative Frequency</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------</td>
<td>---------------</td>
<td>-------------------</td>
<td>------------------------------------------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>22</td>
<td>Auto insurance</td>
<td>Monthly</td>
<td>1</td>
<td>Durable household furnishing and equipment</td>
<td>Monthly</td>
</tr>
<tr>
<td>23</td>
<td>Local public transportation</td>
<td>Monthly</td>
<td>2</td>
<td>New and used motor vehicles (net outlay)</td>
<td>Monthly</td>
</tr>
<tr>
<td>24</td>
<td>Reading</td>
<td>Monthly</td>
<td>3</td>
<td>Tires, tubes, accessories, and other parts</td>
<td>Monthly</td>
</tr>
<tr>
<td>25</td>
<td>Business services</td>
<td>Monthly</td>
<td>4</td>
<td>Recreation and sports equipment</td>
<td>Monthly</td>
</tr>
<tr>
<td>26</td>
<td>Gambling</td>
<td>Monthly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Life and other personal insurance</td>
<td>Monthly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>Prescription drugs (net outlay)</td>
<td>Monthly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>Medical supplies (net outlay)</td>
<td>Monthly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>Medical services (net outlay)</td>
<td>Monthly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>Health insurance</td>
<td>Monthly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>College tuition and fees</td>
<td>Monthly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>Nursery, elementary, and other school tuition and fees</td>
<td>Monthly</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>Educational books and supplies</td>
<td>Monthly</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table A.4. Consumption expenditure shares (quarterly frequency).

<table>
<thead>
<tr>
<th></th>
<th>Outlays (%)</th>
<th>(ND+MG) (%)</th>
<th>Strictly (ND+MG) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outlays</td>
<td>100.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Durables</td>
<td>10.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ND+MG</td>
<td>89.1</td>
<td>100.0</td>
<td></td>
</tr>
<tr>
<td>Memorables</td>
<td>18.6</td>
<td>21.2</td>
<td></td>
</tr>
<tr>
<td>Nondurables</td>
<td>70.5</td>
<td>78.8</td>
<td></td>
</tr>
<tr>
<td>Strictly (ND+MG)</td>
<td>77.0</td>
<td>86.4</td>
<td>100.0</td>
</tr>
<tr>
<td>Strictly memorables</td>
<td>12.9</td>
<td>14.6</td>
<td>17.1</td>
</tr>
<tr>
<td>Strictly nondurables</td>
<td>64.1</td>
<td>71.7</td>
<td>82.9</td>
</tr>
</tbody>
</table>

### Table A.5. Consumption expenditure volatility (quarterly frequency).

<table>
<thead>
<tr>
<th></th>
<th>Had Zeros</th>
<th>Had Spikes</th>
<th>Ave Vol.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outlays</td>
<td>0.000</td>
<td>0.274</td>
<td>0.321</td>
</tr>
<tr>
<td>Durables</td>
<td>0.419</td>
<td>0.911</td>
<td>1.184</td>
</tr>
<tr>
<td>ND+MG</td>
<td>0.000</td>
<td>0.084</td>
<td>0.208</td>
</tr>
<tr>
<td>Memorables</td>
<td>0.028</td>
<td>0.554</td>
<td>0.515</td>
</tr>
<tr>
<td>Nondurables</td>
<td>0.000</td>
<td>0.080</td>
<td>0.199</td>
</tr>
<tr>
<td>Strictly (ND+MG)</td>
<td>0.000</td>
<td>0.088</td>
<td>0.206</td>
</tr>
<tr>
<td>Strictly Memorables</td>
<td>0.053</td>
<td>0.609</td>
<td>0.569</td>
</tr>
<tr>
<td>Strictly Nondurables</td>
<td>0.000</td>
<td>0.078</td>
<td>0.195</td>
</tr>
</tbody>
</table>

#### A.1 Expenditure spikes and inactivity for selected detailed goods categories

In this section, we document the frequency of expenditure spikes and zero purchases for selected memorable goods categories: total expenditure on trips and vacations, clothes and shoes, jewelry and watches. We also report the expenditure spikes and zero purchase patterns for two durable goods categories, new and used vehicles (net outlay), and tires, tubes, accessories and other parts, as a comparison with memorable goods. Figure A.1 shows that most households have at least one consumption expenditure spike within a 12-month period for these selected memorable and durable goods, and the expenditure on these goods tends to be quite lumpy. From Figure A.2, we observe that indeed, memorable goods, as well as durable goods, display infrequent monthly expenditures.
Figure A.1. Number of months with expenditure spikes ($\kappa = 1.5$).
Figure A.2. Number of months with zero purchases.
Appendix B: Quantitative model derivations

Derivation of Euler Equations. The household’s maximization problem is given by

\[ V(M, N, S, z) = \max_{C_{m', S'}} \{ U(C_n, C_{m', M}) + \beta \mathbb{E} V(M', N', S', z') | z \} \]

s.t.

\[ C_n = Y + (1 + r)S - C_{m'} - S', \]
\[ M' = (1 - \delta_m)M + \max\{C_m - N, 0\}, \]
\[ N' = (1 - \rho)N + \rho C_m, \]
\[ S' \geq 0, \]
\[ \ln Y = \bar{y} + z, \]
\[ z' = \rho z + \epsilon. \]

We could rewrite the household’s maximization problem as

\[ V(M, N, S, z) = \max_{N', S'} \{ U(C_n, (N' - (1 - \rho)N)/\rho, M) + \beta \mathbb{E} V(M', N', S', z') | z \} \]

s.t.

\[ C_n = Y + (1 + r)S' - S' - \frac{1}{\rho} (N' - (1 - \rho)N), \]
\[ M' = (1 - \delta_m)M + \frac{1}{\rho} \max\{N' - N, 0\}, \]
\[ S' \geq 0, \]
\[ \ln Y = \bar{y} + z, \]
\[ z' = \rho z + \epsilon. \]

The first-order conditions imply that the following two equations must hold at optimum:

\[ \frac{\partial U}{\partial C_n}(C_n, C_{m'}, M) = \beta \mathbb{E} \frac{\partial V}{\partial S}(M', N', S', z') + \lambda_{S'}, \]
\[ \frac{\partial U}{\partial C_{n'}}(C_n, C_{m'}, M) - \frac{\partial U}{\partial C_{m'}}(C_n, C_{m'}, M) \]
\[ = \mathbf{1}_{C_m > N} \cdot \beta \mathbb{E} \frac{\partial V}{\partial M}(M', N', S', z') + \rho \beta \mathbb{E} \frac{\partial V}{\partial N}(M', N', S', z'), \]

where \( C_n = Y + (1 + r)S - C_{m'} - S', \lambda_{S'} \) is the Lagrange multiplier associated with the borrowing constraint \( S' \geq 0, \) and \( \mathbf{1}_{C_m > N} \) is an indicator function that equals 1 if and only if \( C_m > N. \)

The envelope theorem implies that the following conditions hold at the optimum:

\[ \frac{\partial V}{\partial M}(M, N, S, z) = \frac{\partial U}{\partial M}(C_n, C_{m'}, M) + (1 - \delta_m)\beta \mathbb{E} \frac{\partial V}{\partial M}(M', N', S', z'), \]
The Euler equation for the optimal consumption path of nondurable goods $C_n$ is standard,

$$\frac{\partial U}{\partial C_n}(C_n, C_m, M) - (1 + r)\beta E\frac{\partial U}{\partial C_n}(C_n', C_m', M') = \lambda_S,$$

where $\lambda_S$ is the Lagrange multiplier associated with the borrowing constraint $S' \geq 0$.

Under our utility specification, the Euler equation of $C_{n,t}$ is given by the following equation:

$$C_{n,t}^{\gamma - 1} - (1 + r)\beta E_t C_{n,t+1}^{\gamma - 1} = \frac{\lambda_S}{\xi}.$$

The optimal consumption path of memorable goods $C_m$ rely on not only the borrowing constraint and the interest rate but also the memory stock $M$ and the past experience level of memorable goods consumption $N$,

$$\frac{\partial U}{\partial C_n}(C_n, C_m, M) - \frac{\partial U}{\partial C_m}(C_n, C_m, M)$$

$$= (1 - \rho)\beta E\left(\frac{\partial U}{\partial C_n}(C_n', C_m', M') - \frac{\partial U}{\partial C_m}(C_n', C_m', M')\right)$$

$$+ 1_{C_m > N} \cdot \beta E\frac{\partial V}{\partial M}(M', N', S', z') - \rho \beta^2 E\left(1_{C_m > N'} \cdot E\frac{\partial V}{\partial M}(M'', N'', S'', z'')\right)$$

$$= (1 - \rho)\beta E\left(\frac{\partial U}{\partial C_n}(C_n', C_m', M') - \frac{\partial U}{\partial C_m}(C_n', C_m', M')\right)$$

$$+ 1_{C_m > N} \cdot \beta E\frac{\partial U}{\partial M}(C_n', C_m', M') + 1_{C_m > N} \cdot \beta^2 (1 - \delta_m) E\frac{\partial V}{\partial M}(M'', N'', S'', z'')$$

$$- \rho \beta^2 E\left(1_{C_m > N'} \cdot E\frac{\partial V}{\partial M}(M'', N'', S'', z'')\right).$$

Under our current utility specification, the above equation can be rewritten as

$$C_{n,t}^{\gamma} - \frac{1 - \xi}{\xi} K_{t}^{\gamma} = (1 - \rho)\beta E\left(C_{n,t+1}^{\gamma} - \frac{1}{\xi} K_{t+1}^{\gamma}\right) + 1_{C_m > N} \cdot \beta \xi \frac{1 - \xi}{\xi} E K_{t+1}^{\gamma}$$

$$+ \frac{1}{\xi} 1_{C_m > N} \cdot \beta^2 (1 - \delta_m) E\frac{\partial V}{\partial M}(M_{t+2}, N_{t+2}, S_{t+2}, z_{t+2})$$

$$- \frac{1}{\xi} \rho \beta^2 E\left(1_{C_m,t+1 > N_{t+1}} \cdot E\frac{\partial V}{\partial M}(M_{t+2}, N_{t+2}, S_{t+2}, z_{t+2})\right),$$

where $K_t = C_{m,t} + \xi M_t$. 

\[ \Box \]
Derivation of Euler Equations. Define

\[
\lambda_{n,t} = \frac{\lambda_{S_{t+1}}}{\xi(1 + r)\beta E_t C_{n,t+1}^{\gamma}},
\]

\[
\lambda_{m,t} = \begin{pmatrix}
\left(1 - \frac{(1 - \rho)}{1 + r}\right)\xi C_{n,t}^{\gamma} - \xi \frac{(1 - \rho)}{1 + r} \lambda_{S_{t+1}} - \mathbf{1}_{C_{m,t} > N_t} \beta E_t \frac{\partial V}{\partial M} (M_{t+1}, N_{t+1}, S_{t+1}, z_{t+1}) \\
+ \rho \beta^2 E_t \left[\frac{\partial V}{\partial M} (M_{t+1}, N_{t+2}, S_{t+2}, z_{t+2})\right]
\end{pmatrix}
\]

where \(\xi\) is the risk aversion coefficient. Rational expectations implies that at optimum the following equation must be true:\(^1\)

\[
\frac{(1 + r)\beta C_{n,t+1}^{-\gamma}}{C_{n,t}^{\gamma}} (1 + \lambda_{n,t}) = 1 + e_{n,t+1},
\]

(B1)

\[
\frac{(1 - \rho)\beta [(C_{m,t+1} + \xi M_{t+1})^{-\gamma}] (1 + \lambda_{m,t})}{(C_{m,t} + \xi M_t)^{-\gamma}} = 1 + e_{m,t+1},
\]

(B2)

where \(e_{n,t+1}\) and \(e_{m,t+1}\) can be interpreted as the expectation error, and by construction \(e_{n,t+1}\) and \(e_{m,t+1}\) are uncorrelated with information known at time \(t\). Taking logs on both side and taking a linear approximation\(^2\) of equation (B1), we obtain the linearized Euler equation for nondurable consumption:

\[
C_{n,t+1} - C_{n,t} = \frac{1}{\gamma} \left[\log((1 + r)\beta) + \log(1 + \lambda_{n,t}) - \log(1 + e_{n,t+1})\right].
\]

Note that when the borrowing constraint is not binding at period \(t\) \((\lambda_{S_{t+1}} = 0)\) \(\lambda_{n,t} = 0\). Doing the same with equation (B2) yields

\[
C_{m,t+1} - C_{m,t} = \frac{1}{\gamma} \left[\log((1 - \rho)\beta) + \log(1 + \lambda_{m,t}) - \log(1 + e_{n,t+1})\right] - \xi (M_{t+1} - M_t)
\]

and plugging in the law of motion for \(M_{t+1}\) delivers the linearized Euler equation for memorable consumption expenditures:

\[
C_{m,t+1} - C_{m,t} = \frac{1}{\gamma} \left[\log((1 - \rho)\beta) + \log(1 + \lambda_{m,t}) - \log(1 + e_{n,t+1})\right] - \xi (-\delta_m M_t + \max\{C_{m,t} - N_t, 0\})
\]

In these equations, the constants \(\check{\gamma}, \hat{\gamma}\) are products of the risk aversion coefficient \(\gamma\) and approximation constants.

\[\square\]

Appendix C: Model solution algorithm

The model solution algorithm is as follows:

\(^1\)See Parker and Preston (2005) and Parker (1999) for similar analyses for nondurable goods expenditure.

\(^2\)The linear approximation used here is \(\log y_{t+1} - \log y_t = (y_{t+1} - y_t)/\check{y}\) for some \(\check{y}\).
Step 1. Guess an initial value of value function $V^{(0)}$ at each grid point of the state space, use OLS regression to calculate the Smolyak coefficients associated with value function $V^{(0)}$.

Step 2. At each state space grid point, value function at the $i$th iteration, $V^{(i)}$, is maximized by searching memorable goods consumption $C_m$ over a discrete grid

$$V^{(i)}(M, N, S, z) = \max_{C_m \in \text{Grid of } C_m} \left\{ W^{(i)}(M, N, S, z, C_m) \right\},$$

where $W^{(i)}(M, N, S, z, C_m)$ is the value function associated with memorable goods consumption $C_m$ for given state space variables $(M, N, S, z)$, that is,

$$W^{(i)}(M, N, S, z, C_m) = \max_{S'} \left\{ U(C_n, C_m, M) + \beta \mathbb{E}[V^{(i-1)}(M', N', S', z') | z] \right\}.$$

The solution of optimal savings $S'^*$ associated with memorable goods consumption $C_m$ is characterized by the following equation:

$$-\frac{\partial U(Y + (1 + r)S - C_m - S'^*, C_m, M)}{\partial C_n} + \beta \frac{\partial \mathbb{E}[V^{(i-1)}(M', N', S'^*, z') | z]}{\partial S'} = 0$$

and $S'^* = 0$ if $-\frac{\partial U(Y + (1 + r)S - C_m, C_m, M)}{\partial C_n} + \beta \frac{\partial \mathbb{E}[V^{(i-1)}(M', N', 0, z') | z]}{\partial S'} \leq 0$.

For $(M', N', S', z')$ outside the state space grid, the value of value function $V^{(i-1)}(M', N', S'^*, z')$ is calculated via interpolation using Smolyak coefficients. Furthermore, $\mathbb{E}[V^{(i-1)}(M', N', S'^*, z') | z]$ is calculated using quadratic rule numerical integration method.

Step 3. Update Smolyak coefficients associated with value function $V^{(i)}$.

Step 4. Repeat Steps 2 to 3 until the value of value function at each state space grid point and associated Smolyak coefficients converge.

Appendix D: Computation of stationary distribution

Our model predicts that there is a cross-sectional stationary distribution of state variables. There is no analytical solution to the household's consumption-savings problem, so we characterize the cross-sectional distribution of $(M_t, N_t, S_t, z_t)$ numerically using the Markov chain Monte Carlo (MCMC) simulation method. Specifically our procedure is as follows:

Step 1: At period $t = 0$, we randomly simulate state variables $(M_0, N_0, S_0)$ for each household $h \in \{1, \ldots, H\}$ from an arbitrary initial distribution $F^{(0)}(M, N, S)$, and draw $z_0$ from the distribution $N(0, \sigma^2/(1 - \rho^2))$ for each household.

Step 2: At period $t = 0$, for given state variables $(M_t, N_t, S_t, z_t)$, households optimally make their current memorable goods consumption $C_{m,t}$ and period $t + 1$ savings decisions $S'^{t + 1}_t$. Households' period $t + 1$ state variables $M'^{t + 1}_t$ and $N'^{t + 1}_t$ are updated according to equations (7) and (5), respectively. Households' period $t + 1$ income shock $z_{t + 1}$ is randomly drawn according to the conditional distribution $N(\rho z_t, \sigma^2)$. The updated state variables $(M'^{t + 1}_t, M'^{t + 1}_t, M'^{t + 1}_t)$ for $H$ households yield the numerical distribution $F^{(1)}(M, N, S)$.
Step 3: Check if distribution $F^{(1)}(M, N, S)$ converges to $F^{(0)}(M, N, S)$ by checking whether the mean and variance of the state variable $M, N, S$ are the same under these two distributions. If the distribution has not converged, repeat step 2 for $t = 2, \ldots$. 

**Appendix E: Welfare cost analysis**

**Derivation of Equation (12).** Define as

$$
\tilde{V}(M, N, S; g) = \xi \left[ (1 - g) \tilde{C}_n(M, N, S) \right]^{1 - \gamma} \frac{1}{1 - \gamma}
+ (1 - \xi) \left[ ((1 - g) \tilde{C}_m(M, N, S) + \zeta(1 - g)M)^{1 - \gamma} \right] \frac{1}{1 - \gamma}
+ \beta \tilde{V}(\tilde{M}', \tilde{N}'(M, N, S), \tilde{S}'(M, N, S); g).
$$
Note that
\[ \tilde{V}(M, N, S; g) = (1 - g)^{1-\gamma}\tilde{V}(M, N, S; g = 0) = (1 - g)^{1-\gamma}\tilde{V}(M, N, S). \quad (E1) \]

As for \( \hat{V} \), we can define \( \hat{W}(S; g) \) by
\[ \hat{W}(S; g) = \frac{(1 - g)\tilde{C}_n(S)}{1 - \gamma} + \beta \hat{W}(\hat{S}'(S); g). \]

Note that
\[ \hat{W}(S; g) = (1 - g)^{1-\gamma}\hat{W}(S; g = 0) = (1 - g)^{1-\gamma}\hat{W}(S). \quad (E2) \]

\( \tilde{V}(M, N, S; g) \) is lifetime utility in the no-risk economy with memorable goods, but with nondurable and memorable consumption scaled up by a factor \( g \) at all future dates. The function \( \hat{W}(S; g) \) has a similar interpretation.

For \( \gamma = 1 \), a similar calculation yields
\[ \tilde{V}(M, N, S; g) = \frac{\log(1 - g)}{1 - \beta} + \tilde{V}(M, N, S; g = 0) = \frac{\log(1 - g)}{1 - \beta} + \tilde{V}(M, N, S), \]
\[ \hat{W}(S; g) = \frac{\log(1 - g)}{1 - \beta} + \hat{W}(S; g = 0) = \frac{\log(1 - g)}{1 - \beta} + \hat{W}(S). \]

The welfare cost of consumption fluctuations for a household in state \((M, N, S)\) is then defined (in the model with and without memorable goods, resp.) as the solution to
\[ \tilde{V}(M, N, S; g(M, N, S)) = V(M, N, S, z = 0), \]
\[ \hat{W}(S; g^W(S)) = W(S, z = 0), \]
where setting \( z = 0 \) in the model with risk again assures that households have the same income today and same expected income from tomorrow on in both worlds. Solving for \( g(M, N, S) \) and \( g^W(S) \) gives, exploiting equations (E1) and (E2),
\[ 1 - g(M, N, S) = \left[ \frac{V(M, N, S, z = 0)}{\tilde{V}(M, N, S)} \right]^{\frac{1}{1-\gamma}}, \]
\[ 1 - g^W(S) = \left( \frac{W(S, z = 0)}{\hat{W}(S)} \right)^{\frac{1}{1-\gamma}}. \]

Appendix F: Revisiting an excess sensitivity test of consumption: Data and sample selection
To insure comparability with Souleles (1999) our empirical strategy, as well as crucial sample selection choices and variable definitions, follows his as much as possible. Our definition of nondurable and memorable goods is the same as in previous sections. As discussed in Section 3, our definition of nondurable and memorable goods combined is equivalent to Souleles’s (1999) nondurable goods (ND+MG), and our definition of
strictly nondurable and strictly memorable goods combined equals Souleles’s (1999) definition of strictly nondurable goods (Strictly (ND+MG)).³

The sample was selected in a way that closely follows the selection criteria provided in Souleles (1999).⁴ The CEX asks about tax refunds twice, in a household’s first and final interview. Each time what is recorded is the value of federal tax refunds received by the households in the 12 months before the interview month. Thus the refund variable in the CEX has a reference period of 12 months. About 80% of the refunds were mailed in March, April, and May during the years 1980–1991,⁵ and thus following Souleles (1999), we deflate refunds by the average of the monthly CPI for all items averaged over March, April, and May. All nominal variables were deflated to 1982–1984 dollars.

References


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³The major components of strictly nondurables, defined in Souleles (1999), are food; household operations, including monthly utilities and small-scale rentals; apparel services and rentals; transportation fuel and services; personal services; and entertainment services and high-frequency fees. We further break down the above consumption groups into two consumption categories: strictly nondurable and strictly memorable goods by introducing memorable goods.

⁴A household was dropped from the sample if there were multiple consumer units in the household, or if the household lived in student housing or the head of household was a farmer; a household quarter was dropped if the household lacked basic food expenditure for any month of the quarter, or if any food was received as pay in the quarter. A household quarter is dropped if the age of household head increased by more than one or decreased moving into next quarter. The sample was restricted to households with heads aged 24–64. Finally, a household is dropped if the income report is incomplete or any of the income or financial records are invalid. We thank Nick Souleles for sharing the data appendix of Souleles (1999).

⁵Refer to Table 2 in Souleles (1999).