Testing Ambiguity Theories with a Mean-Preserving Design

BY

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Abstract

Prominent models such as MEU/α-MP and KMM interpret ambiguity aversion as aversion against second-order risks associated with ambiguous acts. We design an experiment where the decision maker draws twice with replacement in the typical Ellsberg two-color urns, but with a different color winning each time. Given this set of mean-preserving prospects, MEU/α-MP, KMM and Savage's SEU all predict unequivocally that risk-averse DMs will avoid the 50-50 urn that exhibits the highest risk conceivable, while risk-seeking ones do the opposite. However, we observe a substantial number of violations in the experiments. It appears that the ambiguity premium is partially paid to avoid the ambiguity issue per se, which is distinct from notions of second-order risk. This finding is robust even when there is only partial ambiguity, and applicable to all models that satisfy a monotonicity condition.

KEYWORDS: Ambiguity, Ellsberg paradox, expected utility, experiment, mean preserving, monotonicity, partial ambiguity, second-order risk, source premium

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1. Introduction

The Ellsberg Paradox refers to the outcome from Ellsberg's (1961) thought experiments, that missing information about objective probabilities can affect people's decision making in a way that is inconsistent with Savage's (1954) subjective expected utility theory (SEU). Facing two urns simultaneously in Ellsberg's two-color problem, one with 50 red and 50 black balls (the risky urn) and the other with 100 balls in an unknown combination of red and black balls (the uncertain or ambiguous urn), most people prefer to bet on the risky urn, regardless of the winning color. This phenomenon is often called ambiguity aversion. Many subsequent experimental studies confirm Ellsberg's finding, as for example surveyed in Camerer and Weber (1992).

The issue of ambiguity has been widely discussed in general applications. Portfolio home bias in favor of domestic securities may be rooted in investors assigning higher ambiguity to foreign titles (Kang and Stulz, 1997; Coval and Moskowitz, 1999; Dlugosch and Wang, 2014). Investors' expected excess returns are higher when information quality is more uncertain (Epstein and Schneider, 2008). Limited stock-market participation by U.S. households is well illustrated as arising from the heterogeneity among uncertainty-averse investors. (Cao, Wang and Zhang, 2005). Uncertainty about future rewards may cause job-searchers to accept less payment at an earlier time than would be optimal otherwise (Cox and Oaxaca, 2000; Oprea, Friedman and Anderson, 2009). Presence of ambiguity is shown to increase the provision of public goods (Eichberger and Kelsey, 2002), and to affect equilibrium prices in oligopoly firms (Eichberger, Kelsey and Schipper, 2009).

Many extensions to SEU have been proposed to rationalize the Ellsberg paradox and the observed ambiguity aversion. Among the most prominent ones, Gilboa and Schmeidler (1989) develop the maxmin expected utility (MEU) theory, where the decision maker (DM) has a set of prior beliefs associated with the ambiguous prospect and assigns the minimal SEU utility based on this set as their MEU utility. It is later generalized to the so-called α-MP (multi-prior) model by Ghirardato, Maccheroni, and Marinacci (2004). MEU solves the paradox and has been applied to studies on asset pricing in Dow and Werlang (1992) and Epstein and Wang (1994) among others.

Another theory that has found broad applications because of its convenient functional form is the smooth model of ambiguity aversion by Klibanoff, Marinacci, and Mukerji (2005, KMM). KMM assumes that the DM is a Savage-type subjective expected utility maximizer, on the space of second-order compound lotteries. Chen, Ju, and Miao (2009), Hansen (2007), Hansen and Sargent (2008), Ju and Miao (2012) and Maccheroni, Marinacci and Ruffino (2013) successfully applied KMM to studies of asset pricing, to obtain internally consistent calibration of ambiguity attitudes and to explain issues such as the equity premium puzzle.

A third is the model of Choquet expected utility (CEU) by Schmeidler (1989), where the DM uses a weighting function called capacity to evaluate prospects.[1] Mukerji and Tallon (2004) survey application of CEU in various areas of economics such as insurance demand, asset pricing, and inequality measurement.

Given the success in the applied fields, many new experimental studies have been conducted to test these models and characterize subjects' behavior accordingly. However, all previous experiments on ambiguity aversion of which we are aware share the feature that the ambiguous prospect can be associated with a lottery that is of either lower mean or higher variance than the benchmark risky prospect. As such, one cannot distinguish whether the observed ambiguity aversion reflects willingness to pay an ambiguity premium for the second-order risk associated with the uncertain act, which α-MP (MEU) and KMM predict, or for the issue of ambiguity per se, which seems to be behind the ideas of source dependence studies initiated by Heath and Tversky (1991) and Fox and Tversky (1995). We present a lottery design, which is a simple modification of Ellsberg's two-color problem that enables this separation.

In our design, the DM draws twice with replacement from an urn with *2N* balls colored red and white. We have the distinctive rule that a different color wins each draw. Winning in one draw yields a prize of x and the payoff is the sum in both draws. In an urn with h red and $2N$-h white balls, the winning chance for the payoff 0, and symmetrically the same for $2x$, is $h(2N-h)/4N^2$, and that for x is $1-h(2N-h)/2N^2$. This design implies the novel and crucial feature that all conceivable color compositions in the urn yield the same expected value

[1]For further theoretical models of multiple priors, second-order sophistication, and rank-dependent utility, see Segal (1987, 1990), Casadesus-Masanell, Klibanoff, and Ozdenoren (2000), Nau (2006), Chew and Sagi (2008), Ergin and Gul (2009) and Seo (2009) among others. Wakker (2008), Eichberger and Kelsey (2009), and Etner, Jeleva and Tallon (2012) offer excellent surveys.

x and differ only in the variance $2x^2h(N-h)/N^2$, which *increases* with the balance of color in the urn. In other words, we restrict ourselves to a mean-preserving class of prospects. The payoff is risk free if all balls in the urn are of the same color. In the experiments, DMs will face both an ambiguity and a (uniform distribution) compound-lottery problem. Due to mean preserving within the ambiguous set of lotteries, according to the well-known theories of SEU, α-MP (MEU) and KMM, a risk-averse DM prefers the ambiguous urn to the 50-50 risky urn; while a risk-seeking DM's preference displays the exactly reversed order. In fact, irrespective of the risk attitude test, these theories with some proper extensions also predict that the DM is to consistently show the same order of preference in the compound lottery problem. For cleaner theoretical predictions, we further design a so-called partial ambiguity treatment (PA) where the color composition in the ambiguity urn is partly unknown. This also serves as a robustness check for our basic finding of persistent violation against those models of ambiguity.

It turns out that, in a conservative estimation by treating all risk-neutral DMs as non-violation cases, 27-52% subjects in different treatments violated the predictions of SEU, α-MP (MEU) and KMM. Disregarding the risk attitude, 23-33% subjects showed inconsistent behavior across the two main decision tasks, in violation of the theoretical prediction. The statistical conclusions are robust over both PA and FA.

These findings present considerable challenges to the dominant approach of interpreting ambiguity aversion as aversion against associated second-order risks. It appears that many subjects are subconsciously willing to pay a premium to avoid the issue of ambiguity per se; i.e. some psychological or neural factors may help shape decisions beyond 2nd-order risk consideration.

In the next section, we discuss the relevant preference models, our experimental design, and the associated theoretical predictions. Data analysis is in Section 3. We then further interpret our results in relation to findings in the literature in Section 4. Section 5 concludes.

2. Experimental Design and theoretical prediction

Decision problems of the experiment

There are three urns labeled B, C, and D. A typical urn has $2N$ balls, each of which can be red or white colored. The novel feature of our design is to have subjects draw from the selected urn twice with replacement, with a different color winning 50 Yuan each draw. If the first draw is red and the second is white, a participant gets 100 Yuan; if the two draws are of the same color, he gets 50 Yuan; but if the two colors are in the order of white first and red second, he gets 0. Urn B is the 50-50 risky one with exactly N red and N white balls. Urn C is the ambiguous urn where any in a set M of up to 2N+1 color compositions maybe the true one. Urn D is one with a uniform compound lottery in which any color composition in the set M has equal chance to be chosen.

Subjects face three simple decision problems one after another. Problem 1 is meant to test their risk attitude. On a list of 20 cases of sure payoffs that range from 5 to 100 Yuan in steps of 5 Yuan, subjects have to choose either the sure payoff or the risky one, Choice B, for every case.[2] Problem 1 is in fact a simple form of the MPL procedure that can also be viewed as a modified version of the BDM procedure.[3] Problem 2 is our main test for theoretical predictions regarding ambiguity aversion. In this problem, subjects have to decide between Choice B and Choice C. Problem 3 is a test of preference over objective compound lotteries, where subjects are to choose between a simple lottery of Choice B and a compound lottery of Choice D.

We have two main treatments. In the *full ambiguity* treatment (FA), there are 10 balls in the urn and any of the 11 possible color compositions can occur in Choice C. In the *partial ambiguity* treatment (PA), there are 16 balls in the urn in Choice C. There exists 17 possible color compositions, but only those compositions can occur where the number difference between the two colors is at least 6, which admits only 12 out of the 17 possibilities. The design is chosen so that the number of admissible compositions is similar (#M=11 in FA vs. #M=12

[2]We aim at revealing individual certainty equivalent values of Choice B. Though we may alternatively replace Choice B with its reduced form (100, 1/4; 50, 1/2; 0, 1/4) here, it would lose the structural congruence to Choice C and D, which we consider crucial to our design.

[3]Sapienza, Zingales and Maestripieri (2009) use a similar method. See Becker, DeGroot, and Marschak (1964) for BDM procedure. See Holt and Laury (2002) for multi-price-list (MPL) procedure. See also Harrison and Rutström (2008) and Trautmann, Vieider and Wakker (2011). Detailed discussion can be found in online appendix.

in PA). We keep the feasible set of priors similar between PA and FA, but the ambiguity range associated with PA is not so small as to make the ambiguity issue irrelevant.[4] The primary purpose of designing two treatments this way is to check whether and how any potential violation of the main hypotheses we will derive next is robust.[5] Note, however, that the feature of a different color winning each round ensures that the mean of the lottery is always 50 Yuan, independent of the color composition in the urn. In fact, all lotteries can be ranked by variance, with Choice B being associated with the highest possible variance.

Table 1: Complete list of feasible lotteries,[6] $N = 5$

	π_0	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8	π_9	π_{10}
Red	0	1	2	3	4	5	6	7	8	9	10
White	10	9	8	7	6	5	4	3	2	1	0
p(0)	0	.09	.16	.21	.24	.25	.24	.21	.16	.09	0
p(50)	1	.82	.68	.58	.52	.50	.52	.58	.68	.82	1
p(100)	0	.09	.16	.21	.24	.25	.24	.21	.16	.09	0
mean	50	50	50	50	50	50	50	50	50	50	50
variance	0	450	800	1050	1200	1250	1200	1050	800	450	0

Theoretical predictions

Let $S = \{rw, rr, ww, wr\}$ denote the set of possible drawing outcomes in the experiment.[7] An act is a mapping $f: S \to \mathrm{X}$ with $\mathrm{X} = \{0, 50, 100\}$ being the space of payoff outcome in our design. Let $p_h^N \in \Delta(S), h = 0,1,\ldots,2N,$ denote the probability distribution on the state space with h red and $2N$-h white balls, while $\pi_h^N \in \Delta(X)$ denotes the corresponding simple

[4]E.g. a PA design with N=6 and #*M*= 10 would make it too similar to FA, while one with N=500 and #*M*=10 would make it feel almost like a decision under risk without much ambiguity.

[5]Note that our main concern in the PA treatment is, first, to get away from indifference in the theory prediction that is not feasible under FA, and second to serve as a robustness check for FA. For a more systematic experimental investigation of partial ambiguity, see Chew, Miao and Zhong (2013).

[6]Table 1 summarizes all possible first-order lotteries given this payoff rule, with π_h coding for the lottery with h red balls and 10-h white balls. There are exactly 11 of them. Each column lists the distribution of monetary outcome, its mean and its variance. For example, the urn with 4 red and 6 white balls, π_4, gives us the probabilities of .24, .52, and .24 to earn the prize of 0, 50, and 100 Yuan, respectively; with a mean of 50 Yuan and a variance of 1200. Obviously, our modified Ellsberg risky prospect, π_5, has the highest variance of 1250, while all color compositions yield the same mean payoff.

[7]Epstein and Halevy (2013) also use a similar state space.

lottery on the outcome space. Due to our symmetrical design, {h-red, (2N-h)-white} and {(2N-h)-red, h-white} urns induce equivalent prospects. Generically, the probabilities on outcomes are $\pi_h^N(0) = \pi_h^N(100) = \frac{h(2N-h)}{4N^2}$ and $\pi_h^N(50) = 1 - 2\pi_h^N(0)$ respectively. As illustration, Table 1 summarizes the statistical characteristics of all physically feasible simple lotteries in our design, for $N = 5$. The design makes the mean for π_h^N the same 50 for all h, but the variance, $\text{var}\,\pi_h^N = 2 * 50^2 h(N-h)/N^2$, increases from h= 0 to h=N and then symmetrically decreases from h=N to h= 2N, with $\max_h \text{var}\,\pi_h^N = \text{var}\,\pi_N^N = 1250$. The crucial feature for our design is that a more color-balanced urn constitutes a mean-preserving spread to a less balanced one.

Next, we consider the most popular theories on ambiguity and discuss their implications in our design. Savage's (1954) theory assumes that there is a monotone utility function on the outcome space, $u: X \to \mathbb{R}$, such that for each act f, there is a subjective belief $p \in \Delta(S)$ so that the DM has the subjective expected utility (SEU)

$$\text{(1)} \qquad \text{SEU}(f) = \sum_{s \in S} p(s)u(f(s))$$

Ghirardato et al. (2004) have the so-called α-Multi-Prior (α-Maxmin) model, as follows. Given $K \subseteq \Delta(S)$ compact,

$$\text{(2)} \qquad \alpha\text{-MP}(f) = \alpha \min_{p \in K} \sum_{s \in S} p(s)u\big(f(s)\big) + (1-\alpha) \max_{p \in K} \sum_{s \in S} p(s)u\big(f(s)\big)$$

This model is indeed a generalization of the MEU model proposed by Gilboa and Schmeidler (1989). In the extreme case of $\alpha = 1$, (2) induces the original MEU expression as follows.

$$\text{(2a)} \qquad \text{MEU}(f) = \min_{p \in K} \sum_{s \in S} p(s)u(f(s))$$

For simplicity of theoretical proofs, we will proceed with the equivalent formulation on the outcome space with the corresponding $\pi_h^N \in \Delta(X)$ instead. Now, let $c_h := \pi_h^N(0)[u(0) +$

$u(100)] + \left(1 - 2\pi_h^N(0)\right) u(50)$ denote the expected utility given lottery π_h^N for any $h, h' \in \{1,2, \dots 2N\}$,

(3) $\quad c_h - c_{h'} = (\pi_h^N(0) - \pi_{h'}^N(0))[u(0) + u(100) - 2u(50)]$

For any $h' \neq N$, since $\pi_N^N(0) > \pi_{h'}^N(0)$, $c_N - c_{h'} \lesseqgtr 0$ iff $u(0) + u(100) - 2u(50) \lesseqgtr 0$, i.e. iff $\mathrm{CE} \lesseqgtr 50$.[8] In FA treatment of our design, since π_5^5 with the last variance presents a mean preserving spread to any lottery $\pi_h^5 (h \neq 5)$, both SEU and α-MP predict weak preferences of Cover B for a risk-averse DM as well as B over C for risk-seeking DMs. Indifference between C and B may occur in SEU if the chosen belief p happens to be the one corresponding to π_N^N, and in α-MP if in addition $\alpha = 1$. Therefore, for both SEU and α-MP (MEU), choices in FA satisfy the following condition,

(4) $\quad \mathrm{CE} < 50 \Rightarrow \mathrm{B} \precsim \mathrm{C}$ and $\mathrm{CE} > 50 \Rightarrow \mathrm{C} \precsim \mathrm{B}$.

Comparatively, since the design rules out π_8^8 as a candidate for the subjective belief for urn C in PA, both SEU and α-MP predict strict preferences of C over B for a risk-averse DM as well as B over C for risk-seeking DMs, i.e.,

(4a) $\quad \mathrm{CE} < 50 \Leftrightarrow \mathrm{B} \prec \mathrm{C}$ and $\mathrm{CE} > 50 \Leftrightarrow \mathrm{C} \prec \mathrm{B}$.

Note that there exists no ambiguity at all on urn D. As the α-MP model says nothing about compound lotteries, we assume DMs have von-Neumann-Morgenstern preferences for decision under objective risks. By the reduction of compound lottery axiom, we conclude for Problem 3,[9]

(4b) $\quad \mathrm{CE} < 50 \Leftrightarrow \mathrm{B} \prec \mathrm{D}$ and $\mathrm{CE} > 50 \Leftrightarrow \mathrm{D} \prec \mathrm{B}$.

Now, the smooth model of ambiguity aversion (KMM) by Klibanoff et al. (2005) assumes that there is a monotone function $v: \mathbb{R} \to \mathbb{R}$, with which the DM evaluates the expected utility associated with first-order beliefs. For each act *f,* there is a second-order subjective belief $\mu \in$

[8]DM's risk attitude associated with u is revealed in Problem 1 as either risk averse, risk neutral or risk seeking (corresponding to certainty equivalent of first-order risk; CE < 50, = 50, or > 50, i.e., $\mathrm{u}(0) + \mathrm{u}(100) - 2\mathrm{u}(50) < 0, = 0,$ or > 0)

[9]Note for any compound lottery $y = (p_n : z_n)_n$, $\mathrm{var}\, y = \sum p_n \mathrm{var}\, z_n + \sum p_h (\bar{z}_n - \bar{y})^2$. Due to mean preserving, the second term vanishes in our design. The variance for Choice D is 750 in FA and 602 in PA.

$\Delta^2(\mathrm{S})$ so that

$$(5) \qquad \mathrm{KMM}(f) = \int_{\Delta(\mathrm{S})} v\left(\sum_{s\in\mathrm{S}} p(s)u(f(s))\right)\mathrm{d}\mu(p)$$

Note, KMM can be viewed applicable to compound lotteries. In fact, urn D can be interpreted as equivalent to imposing a uniformly distributed second-order distribution in place of μ, in both treatments, the admissible range of which is effectively restricted by our lottery design.[10] For any strictly increasing $v(.)$ and any $\mu \in \Delta^2(\mathrm{S})$, we hence conclude from the definition of KMM for the treatment FA that

$$(6) \qquad \mathrm{KMM(B)} \leq \mathrm{KMM(C)} \Leftrightarrow \mathrm{KMM(B)} < \mathrm{KMM(D)} \Leftrightarrow \mathrm{CE} < 50$$

The proof is straightforward in that, due to monotonicity, $v(c_N)$ is either the maximum or the minimum on $\{c_h: h \in \{1, \ldots, 2N\}\}$, depending on whether the DM is risk seeking or averse. However, it also cannot rule out indifference between urn B and C when the DM evaluate π_5^5 as the only possible prior in urn C, i.e. $\mu(p_5^5) = 1$. Comparatively, PA rules out such possibility of $\mu(p_8^8) = 1$, therefore, the KMM's prediction for PA is that

$$(6a) \qquad \mathrm{KMM(B)} < \mathrm{KMM(C)} \Leftrightarrow \mathrm{KMM(B)} < \mathrm{KMM(D)} \Leftrightarrow \mathrm{CE} < 50$$

In summary, we have the following theoretical predictions to test for our experiment.

Hypothesis 1 (1) *In Problem 2, SEU, α-MP, MEU, and KMM predict that risk-averse individuals with CE<50 in Problem 1 weakly (strictly) prefers C over B in the FA (PA) treatment, while the prediction is exactly reserved for risk-seeking subjects.* (2) *In Problem 3, standard theory prediction has risk-averse DMs strictly preferring D over B in both FA and PA.*

Note that indifference is merely a degenerate case under FA. However, the introduction of the PA treatment enables the strong prediction for Problem 2 not possible under FA. Note also that any decision in Problems 2 and 3 by a Problem-1 risk-neutral individual is trivially consistent with the theory prediction, as is obvious from equation (3) above. Besides, since the theories

[10]The variance of all available simple lotteries, as presented in Table 1, limits the variance of compound lottery to the interval [0,1250] in FA, and [0, 1172] in PA.

predict that people with non-neutral risk attitudes should have a strict preference in both Problems 2 and 3, it becomes redundant to provide the option of indifference between the two choices in the design. As an implication of Hypothesis 1, we note that regardless of Problem 1, the DM is to make consistent choices BB (if risk seeking) or CD (if risk averse) in Problem 2 and 3. This yields the following hypothesis for testing.

Hypothesis 2 (Weak consistency) *To be consistent with models of SEU, α-MP, MEU and KMM, individuals shall choose either BB or CD in Problems 2 and 3, weakly so in FA and strictly so in PA.*

In contrast, such sharp behavior predictions cannot be made in the spirit of Choquet expected utility (CEU), another popular model by Schmeidler (1989), closely related to the rank-dependent utility (RDU) model developed by Quiggin (1982). Assume finite partition for the state space S into events $E_1, E_2, \dots, E_n$, such that the associated outcomes via an act f are completely ordered, with $x_1 \geq x_2 \cdots \geq x_n$. Under CEU, the DM evaluates u with a capacity w, i.e. a weighting function defined on the sigma algebra generated by the partition $\{E_i\}_{i=1}^n$, as follows,[11]

$$\text{(7)} \qquad \mathrm{CEU}(f) = \sum_{i=1}^{n} \left[w\left(\bigcup_{j=1}^{i} E_j \right) - w\left(\bigcup_{j=1}^{i-1} E_j \right) \right] u(x_i)$$

Let $q(x_i) := w\left(\bigcup_{j=1}^{i} E_j\right) - w\left(\bigcup_{j=1}^{i-1} E_j\right)$, it follows that $q(x_i) \geqq 0$ for all i and $\sum_{i=1}^{n} q(x_i) = 1$, i.e. q is a probability distribution; and, similar to SEU in appearance, equation (7) can be rewritten as $\mathrm{CEU}(f) = \sum_{i=1}^{n} q(x_i) u(x_i)$.[12] Note that the weighting function w by definition is not subject to much restriction, nor is the virtual lottery q; particularly q is not necessarily in the mean-preserving class imposed in our design. Such room of freedom, however, is impossible under SEU, MEU, α-MP and KMM. In our experimental study, this added degree of freedom proves to be crucial to distinguish CEU from the others.

[11] A weighting function w is a capacity generated by the events, if it is non-negative, $w(\emptyset) = 0$, $w(S) = 1$, and $w(A) \leq w(B)$ whenever $A \subseteq B$.
[12] We refer the reader to Wakker (2008) for more detailed discussion.

Lemma 1 *For any combination of decisions in Problems 1 and 2 in FA or in PA, there is a* weighting function w *under the CEU model that rationalizes them*.

A formal proof of Lemma 1 can be found in online appendix. Note CEU was primarily developed for decision under ambiguity. For decision under objective risk such as for Choices B and D, it refers to the standard expected utility treatment the same way SEU and MEU do. In this sense, CEU induces the same prediction regarding Problem 3 as the latter. Since the focus of our study is ambiguity, we elect not to elaborate further.[13]

Experimental procedure

Our instructions were done with a PowerPoint presentation. Subjects were to hand in their decisions on one problem before they received instructions for the next one. To increase credibility and comprehension, we demonstrated drawings with the urn to be used later in Choices B and D during instructions. The Choice C urn was prepared before the session and placed on the counter for all to see.[14] After all decision sheets were collected, subjects were called upon to have their decisions implemented one by one.[15] We have two versions of FA, denoted FA1 and FA2 respectively. For both PA and FA1, subjects drew randomly from one of the three decision problems and were paid in cash according to the realization of their decisions in that problem. In FA2, we add on other auxiliary tasks after Problems 1-3 that include incentivized comprehension tests and questionnaires, with the implementation procedure extended accordingly. We also have a session for additional incentivized comprehension tests. A detailed summary of all relevant sessions can be found in Table A1 in online appendix.

A total of 426 subjects from Shanghai University of Finance and Economics participated in the study. All participants were first-year college students of various majors ranging from economics and management to science and language. Our data analysis focuses on the main

[13]See Halevy (2007, page 512) for an excellent summary on this issue. Online appendix also offers a proof that the related RDU model can rationalize any decision combination in our Problems 1-3.

[14] Note, our double-draw, alternate-color-win design conceptually removes the subjects' fear of possible manipulation of color composition by the experimenter. Nevertheless, students still regularly asked to inspect the content of the ambiguous urn C after the decision implementation.

[15]After subjects handed in their decisions, they were given the option to have the payment procedure implemented later in the experimenter's office, if they did not wish to wait. Only two of them made use of this option.

treatments PA, FA1 and FA2 with 246 participants; all of them were paid.[16] Average payoff in the three treatments was 60.3 Yuan, and average duration for a session was 40 minutes.[17]

3. Experimental Results

Problem 1 elicits individuals' risk attitudes. The certainty equivalent value (thereafter CE) of the risky lottery (Choice B) in our experiment is defined as the lowest value at which one starts to prefer the sure payoff to the lottery. Overall, 94.72% of the subjects in PA, FA1, and FA2 revealed monotone behavior of switching from B to A with increasing sure payoffs. Subsequent analyses are restricted to these samples only.[18] Note our incentivized comprehension tests show that subjects from the cohort have no problem understanding the statistical implications of our double-draw lottery design.[19] Details are in online appendix.

Table 2 summarizes the mean and standard deviation (std.) of the CE, as revealed in Problem 1, and its distribution over the three basic types of risk attitudes, for each treatment. Note that FA2 has more risk-averse, but fewer risk-neutral people. As we will show later, there is no difference between decisions in FA1 and FA2. Figure B1 in online appendix shows the distributions of subjects' CE values.

Table 2. Statistics on risk attitude from Problem 1

	PA	FA1	FA2
Obs.	77	72	84
Average CE (std.)	50 (12.64)	49.65 (11.11)	47.86 (10.45)
Risk-averse: CE<50	28.57%	34.72%	47.62%
Risk-neutral: CE=50	42.86%	34.72%	21.43%
Risk-seeking: CE>50	28.57%	30.56%	30.95%

[16]We ran a pilot test, the same as the treatment FA1 with randomly paying only about 10% subjects out of 150 subjects, and we also ran an auxiliary session with 30 subjects using incentivized comprehension tests Quiz1 and Quiz2. Details on its motivation, design and outcomes can be found in online appendix.

[17]Note that 1 USD = 6.8 Yuan. Regular student jobs paid about 7 Yuan per hour, and average first jobs for fresh graduates paid below 20 Yuan per hour. The duration of 40 minutes is the average time spent by all subjects including the long waiting time for payoff implementation.

[18]Only 8 out of 85 subjects (9.41%) in the PA treatment, 3 out of 75 subjects (4%) in the FA1 treatment, and 2 out of 86 subjects (2.33%) in the FA2 treatment switched back from A to B, which is deemed anomalous and excluded from our data analysis.

[19]FA2 has a quiz test in the design. Only 5.95% of the subjects (5 out of 84) answered partly incorrectly. We also have the same quiz in a separate session; 6.67% of the subjects (2 out of 30) failed to answer correctly. More discussion can be found in online appendix.

We now turn to testing our hypotheses. As summarized in Hypothesis 1, for risk-averse (seeking) individuals in Problems 2 and 3, the theories predict the choice of C and D (B and B), respectively. Figure 1 reports the rejection rate for Hypothesis 1 separately for risk-averse and risk-seeking DMs, for different treatments. The two-sample test of proportions shows no significant difference in Problems 2 and 3 choices, among risk-averse subjects in both FA1 and FA2 (p=0.7830 and p=0.2367) or among risk-seeking subjects (p=0.8628 and p=0.7904). Note, subsequently, all p-values refer to this test unless noted otherwise. Consider behavior distribution over the 3 risk categories, we find no significant difference between FA1 and FA2, in Problem 2 or 3 behavior (p=0.729 and p=0.121, χ^2 test). Thus, for convenience and clarity, we subsequently merge FA1 and FA2 data into one, called FA, unless noted otherwise. As reported in Table 3, the violation rate varies between 27.27% and 52.08% over different problems and treatments, with a lower bound of the corresponding 95% confidence interval (CI) ranging from 10.73% to 37.19%.[20]

Pooling all subjects in a treatment together, and conservatively treating all risk-neutral DM's decisions as consistent with the theories, the violation rate still ranges from 22.08% to 31.41% with an associated 95%-CI lower bound ranging from 13.42% to 24.22%. Furthermore, 28.57% ([18.85, 40.00]), 43.31% ([34.45, 50.46]) of DMs in PA and FA respectively, violate Hypothesis 1 in at least one of Problems 2 and 3.[21] Note that PA may induce less violation than FA due to a smaller range of ambiguity by design. In fact, this conjecture is correct in a significant manner for risk-averse DMs in Problem 2 (p=0.0601, one-sided).

[20]As visible in Figure 1, the violation rate between risk-averse and risk-seeking subjects are not significantly different (p=0.1335 in Problem 2 of FA, p=0.1216 in Problem 2 of PA and p=0.1216 in Problem 3 of PA) except for in Problem 3 of FA (p=0.0271).

[21]These straightforwardly result from calculations based on the raw data in Table B2 in online appendix, just as almost all other statistics discussed in this section.

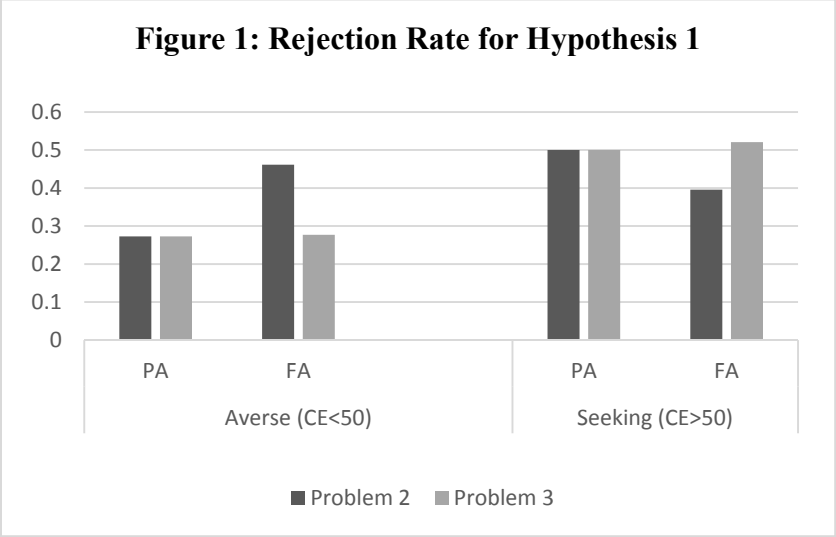


Table 3. Rejection rate for Hypothesis 1 and confidence interval

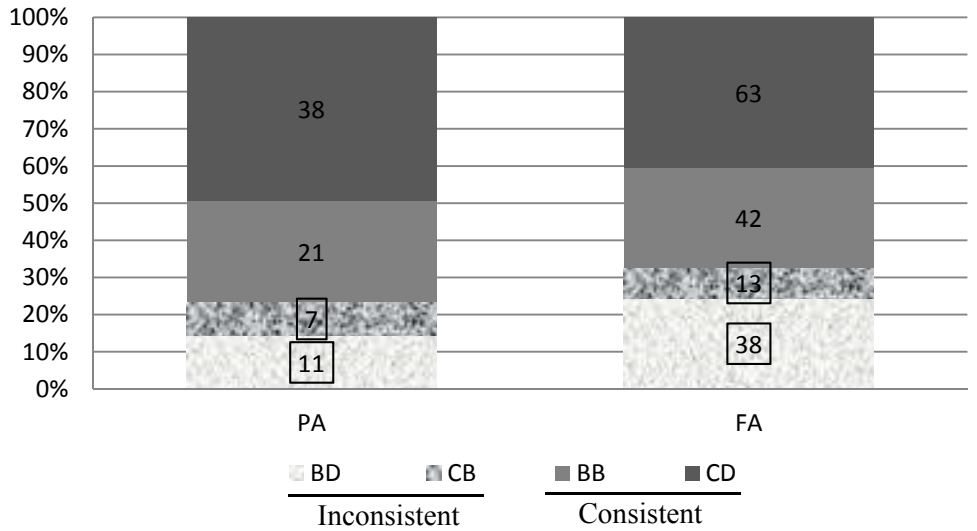
		Obs.	Problem 2		Problem 3	
			Violation (%)	CI	Violation (%)	CI
PA	Risk-averse	22	27.27	[10.73, 50.22]	27.27	[10.73, 50.22]
	Risk-seeking	22	50	[28.22, 71.78]	50	[28.22, 71.78]
	Risk-neutral[a]	33	0	-	0	-
	all pooled	77	22.08	[13.42, 32.98]	22.08	[13.42, 32.98]
FA	Risk-averse	65	46.15	[33.70, 58.97]	27.69	[17.31, 40.19]
	Risk-seeking	48	39.58	[25.77, 54.73]	52.08	[37.19, 66.71]
	Risk-neutral[b]	43	0	-	0	-
	all pooled	156	31.41	[24.22, 39.32]	27.56	[20.72, 35.28]

Note: Pairs of numbers in square brackets [-, -] refer to 95% confidence intervals defined by percentage. All risk-neutral DMs count as non-violation in the "all pooled" category. a: the numbers of choice B are 21 and 14 in Problems 2 and 3 respectively. b: the numbers of choice B are 15 and 11 in Problems 2 and 3 respectively.

As set forth in Hypothesis 2 (weak consistency), the theories can be seen as having a clear prediction on joint decisions within Problems 2 and 3, even without Problem 1 to explicitly elicit the risk attitude. The distribution of all possible decision combinations in Problems 2 and 3, namely BB, BD, CB and CD, is illustrated in Figure 2. Numbers in each column add up to the total sample size in a treatment, while additional square boxes indicate observations of inconsistent behavior that violate Hypothesis 2. The proportions of inconsistent types (BD and CB) are 23.38% ([14.48, 34.41]) and 32.68% ([25.41, 40.65]) in PA and FA, respectively. Such large-scales of inconsistency further suggests that people may inherently treat the ambiguous

and the compound-risk issues differently.[22] The inconsistency rate in PA is lower than that in FA, at a mild significance level of p=0.0885. Note that, this is compatible with the conventional belief that people behave more consistently when facing less ambiguous situations. Note, from risk-averse to risk-neutral and risk-seeking subjects, the violation rates are (40, 25.58, 29.17) for FA and (27.27, 24.24, 18.18) for PA, which seems to suggest that the inconsistency may decline from risk averse to non-averse subjects and from FA to PA in general.[23] In combination with Figure 1, the latter observation suggests that violations against theories of SEU, MEU and KMM might decrease with reduction of ambiguity, such as from FA to PA in our design.

Figure 2: Distribution of decision combinations in Problems 2 and 3



Another pattern of behavioral inconsistency, not directly related to our main hypotheses, is reflected in the relative frequency of people switching actions from B in Problem 2 to non-B in Problem 3, and vice versa. In FA, we find that the switch rates are BD/(BD+BB) = 47.5% ([36.21, 58.98]) and CB/(CB+CD) = 17.11% ([9.43, 27.47]). In PA, the switch rates are BD/(BD+BB) =34.38% ([18.57, 53.19]) and CB/(CB+CD) = 15.55% ([6.49, 29.46]). In both treatments the former is significantly higher than the latter (p=0.0001 and p=0.0545). In FA, the odds of inconsistency are 4.6 times as large if B rather than C is chosen in Problem 2, while

[22]The results from the comprehension tests as reported in online appendix rule out the concern that the statistical implications of our double-draw design may be too difficult for the subjects to understand.
[23]Detailed statistics of the types BB, BD, CB and CD by risk attitudes can be found in Table B2 in online appendix.

in PA, the odds are 2.84 times as large if B is chosen in Problem 2. Thus, it is interesting to observe that in both treatments people with a preference for the ambiguous option in Problem 2 turn out to be more consistent than those with Choice B. One way to understand this result is to think of the decision for C or D as carrying with itself some sort of biased selection for DMs that are more predisposed to follow second-order risk models.

We have strong evidence against the hypothesis that subjects' choices may be random due to potential comprehension or attention issues. For Problem 2, for example, choices by risk averse subjects in PA reject the randomization hypothesis (p=0.0524, binomial test).[24] The McNemar test rejects the hypothesis that BB and CD in FA are chosen equally likely as CB and BD (p=0.0005). Moreover, the χ^2 test shows that the distribution of combined choices across both Problems 2 and 3 is significantly different from randomization for both PA and FA (p <0.001).

4. Discussion

Following SEU, MEU and KMM, ambiguity aversion is traditionally interpreted as willingness to pay a premium to avoid the variability of the range of ambiguity behind the prospect, i.e. a premium to avoid the additional, second-order risk beyond that attached to any single objective distribution. However, when the ambiguous prospect is only associated with mean-preserving contractions over the risky one, thus without any reason to pay a premium based on a wider range of unwanted risks, as in our design, a substantial share of subjects still chose to avoid the ambiguous prospect, in violation of predictions by MEU and KMM.

Failure of a basic monotonicity condition

In fact, our design has implications beyond testing specific ambiguity models such as MEU and KMM. In a general setup such as in Gajdos et al. (2008), suppose the agent has preferences over pairs (f, K) where f is an act and K is a set of priors. A seemingly innocuous *monotonicity* condition is that if (f, p) is preferred to (f, q) for all selections $q \in K$, then (f, p) is also preferred to (f, K); and if (f, q) is preferred to (f, p) for all selections $q \in$

[24] Since the instruction for FA is even simpler with the same level of payoffs, there is no reason to believe in random choice by FA subjects either. Note that incentivized quiz test shows that subjects were aware of the statistical implications of the urns, and they also revealed fair justification for their choices in the questionnaire.

K, then (f, K) is also preferred to (f, p). Let p be the 50-50 urn, and K the ambiguous urn in our design. Risk-averse DMs prefer any $q \in K$ to p, yet many of them violated the monotonicity condition and chose (f, p) over (f, K) in our Problem 2.[25]

Source dependence and neural-imaging studies

It turns out that any outcome in our design is compatible with CEU. In fact, $50 - u^{-1}(\sum_{i=1}^{n} q(x_i)u(x_i))$ can be roughly interpreted as the *source premium*, which would be zero under α-MP/KMM in similar terms due to our mean-preserving prospect design. CEU admits enough maneuverability to find a virtual lottery q outside the admissible, mean-preserving set to evaluate the ambiguous Choice C, and thus explains these violations which MEU and KMM cannot account for. Technically, a CEU DM may overweight the x=0 event as if he is willing to pay a premium to avoid the issue of ambiguity per se, even when it implies nothing but a mean-preserving contraction over the simple-risk prospect.[26]

Note that Al-Najjar and Weinstein (2009) suggest that ambiguity aversion is best understood as individuals' perception that they are in a game situation with the experimenter, who can manipulate the odds against their interests. Stecher et al. (2011) discuss ways to avoid this kind of informational asymmetries between subjects and experimenters in lab studies, while Quiggin (2007) develops a related formal model. Our draw-twice design, however, fully removes subjects' manipulation concern. We nevertheless observed a significant amount of ambiguity aversion.

The interpretation of the DM's willingness to pay a premium for the issue per se in cases of observed violations in our study is generally compatible with the source-dependence interpretation of ambiguity and recent neuroimaging studies in the literature. Many studies feature natural events in their design of ambiguous prospects, also eliminating concerns of manipulable odds, and find that decision under uncertainty depends not only on the degree of uncertainty but also on its source. Heath and Tversky (1991), e.g., find people willing to pay a significant premium to bet on their own judgments instead of on domains over which they feel

[25]We are indebted to an anonymous referee for suggesting this interpretation.
[26]Note, Ellsberg's three-color problem and Machina's (2009) reflection example also involve restricted ambiguity, though not of the mean-preserving type like ours. And whenever this occurs, it may be harder to find a falsifying test for CEU compared to MEU and KMM due to the additional room for maneuvering mentioned above.

lack of competence, which cannot be wholly explained by aversion to ambiguity as asserted in the second-order preference theories, because judgmental probabilities are more ambiguous than chance events. Further corroborating evidence over a wide range of experimental designs can be found in Fox and Tversky (1995), Tversky and Fox (1995), Tversky and Wakker (1995), Fox and Tversky (1998), Fox and Weber (2002), and Chow and Sarin (2002).

Recent neuroimaging studies compare brain activation of people who choose between ambiguous versus risky options and suggest that these two types of decision making rely on different brain mechanisms and processing pathways. For example, evidence in Hsu, Bhatt, Adolphs, Tranel and Camerer (2005) suggests that, when facing ambiguity, the amygdala, which is the most crucial brain part associated with fear and vigilance, and the orbitofrontal cortex (OFC) activates first and deal with missing information independent of its risk implications.[27] Chark and Chew (2012) also find that activity in the amygdala and OFC are positively correlated with the level of ambiguity associated with the decisions. All of these suggest that DMs may become much less probabilistically sophisticated, when the brain switches modes between facing different sources of ambiguity, so much so that DMs may be willing to pay a source premium to avoid the switch. This is also consistent with findings in psychological studies that in general identify multiple processes (some more effortful and analytic, others automatic, associative, and often emotion-based) in play for decisions under risk or uncertainty (Weber and Johnson, 2008). Note that all findings discussed in this subsection are also compatible with the general heuristics explanation that Al-Najjar and Weinstein (2009) extensively discussed.

Evidence about comprehension on ambiguous urn

At the end of treatment FA2, we asked subjects about the reasons for their decision in the ambiguity Problem 2. We handed out different questionnaires for Choice B and C. Option 1 relates to their concerns on the added uncertainty, i.e. ambiguity, attached to Urn C, while option 2 elicits whether the DM is aware of the risk implications in Problem 2 and makes it a relevant reason for decision. Interestingly, among the 42 subjects with the choice B, 31 and 17 subjects chose the options 1 and 2 respectively, while in contrast among the 42 subjects with

[27]See also Huettel, Stowe, Gordon, Warner and Platt (2006), Chew, Li, Chark and Zhong (2008), and Dolan (2007).

the choice C, 14 and 31 subjects chose the options 1 and 2 respectively. We observe a clear trend that preference for B is more likely related to avoiding ambiguity, and preference for C is more motivated by risk consideration.[28] Instruction details and further data summary are in online appendix.

Further robustness check for risk elicitation: Problem 4 in FA2

Our choice of Problem 1 as the method to elicit subjects risk attitude, although often used in ambiguity studies, is not the only risk test conceivable. Besides detailed discussion of alternatives in online appendix, we consider an additional test, incorporated into FA2 as Problem 4, where the DM has to make decision between the urn B and another urn E that is known to consist of one red and nine white balls. The crucial difference to Problem 1 is that there is no certainty prospect involved, which according to prospect theory may induce different risk attitudes. Design details and data summary can be found in online appendix where we link Problem 1, 2 and 4 decisions with one another. In Problem 4, we found that 92.5% (37 out of 40), 94.44% (17 out of 18), and 50% (13 out of 26) of risk-averse, risk-neutral and risk-seeking subjects respectively chose E. Risk-averse subjects have significantly stronger preference for E than risk-seeking ones (p=0.000, χ^2 test). Overall, subject behavior in Problem 1 and 4 seem to be consistent. We name the four decision combinations in Problem 2 and 4 BB, BE, CB and CE separately. Regardless of risk attitude, we find that the switch rates with BE/(BE+BB) = 78.57% ([63.19, 89.70]) and CB/(CB+CE) = 19.05% ([8.60, 34.12]) are significantly different (p=0.0000). This is in fact consistent with our new poll results that many chose B to avoid information uncertainty, while preference for C in the same Problem 2 was more likely linked to statistical calculation.

Other studies on estimation and interpretation of ambiguity models

Given the success of the prominent models of ambiguity in economic applications, many experimental studies have attempted to estimate how well they fit lab data. For example,

[28]This does not mean that subjects who chose B do not understand the draw-twice design. Rather, in terms of Kahneman (2011), the ambiguity problem may be more likely to activate the emotion-related decision heuristics in the brain than the calculation-based circuits. In the comprehension quiz after Problem 4, 79 out of 84 subjects correctly recognized the associated risk issues.

Halevy (2007) tests the preference models for consistency and finds substantial support for KMM (35%) and CEU (35%). Abdellaoui, Baillon, Placido and Wakker (2011) find support for the source preference hypotheses with the CEU model. Baillon and Bleichrodt (2014) find strong evidence against uniform ambiguity aversion in favor of models that assign different ambiguity attitudes to gain and loss domains. Ahn, Choi, Gale and Kariv (2014) find that in a portfolio choice experiment with individuals' heterogeneous preferences, most subjects' behavior is better explained by kinked than by smooth models. Also see Choi, Fisman, Gale and Kariv (2007) for a related study. However, Conte and Hey (2012) find evidence in favor of KMM as the best fitting one among two-stage models in a compound lottery design. Further tests on the α-MP model include Chen, Katuscak and Ozdenoren (2007) and Hayashi and Wada (2011).While these studies are designed to discriminate among different common ambiguity models such as MEU and KMM for their respective predictive power, ours focuses on a test to challenge most such models, except for CEU, in a bundle without further discrimination. Note that, although our design seems to suggest CEU as a more general model, empirical studies may favor the more restrictive alternatives due to issues like over-fitting.

Also, Andersen, Fountain, Harrison, and Rutström (2009) using model variations by Nau (2006) show that subjects behave in an entirely different qualitative way towards risk than towards uncertainty, which is consistent with our findings here.

5. Conclusion

Empirical paradoxes that challenge the existing prominent models, such as Allais (1953) and Ellsberg (1962), have been crucial for the development of decision theories. In this study, we designed an experiment to test the currently prevailing conceptualization of ambiguity as second-order risk, which is shared by prominent models including MEU and KMM. By restricting attention to a mean-preserving class of prospects, the new design allows us to separate people who are avoiding ambiguity per se from those avoiding second-order risk. We found that a substantial proportion of participants violated predictions of this class of models, or more generally a monotonicity condition, without any concern for experimenter manipulation of odds. However, it remains an open issue for future studies to develop a

falsifiable theory that fully integrates the observed source premium for the issue per se with the, simultaneously existing, ambiguity premium for second-order risks. In some sense, behavior in real-world decision-making situations may be pointedly manipulated by either priming them into aversion to ambiguity per se or explicitly training them into thinking of second-order risks.

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