A Extensions and Robustness

A.1 Time-varying copula

While our baseline algorithm treats the copula as time fixed, the method we use to approximate time variations in the value/policy functions can also be applied to the copula framework. For this purpose, we determine the steady-state copula C and its pdf dC. In principle, we then obtain a DCT of this pdf, Θ_{dC}, to determine the important coefficients and proceed just as we did for the value/policy functions.

However, the fact that C is a copula such that its discretized version is defined on a grid of marginal percentiles complicates the setup slightly because integrating out (in practice: summing over) the other dimensions, the copula always needs to reproduce the marginal distributions that are its arguments, i.e.

$$\int_{\mu_{-i}} dC(\mu_i, \mu_{-i}) = d\mu_i \tag{24}$$

must hold. Expressed differently, allowing the perturbation of the entire Θ_{dC} produces too many degrees of freedom. Therefore, replacing the functionals with their discrete analogues, we do not calculate the DCT of the entire array dC but leave out the last entry along each dimension. We can then freely perturb these coefficients and reconstruct the perturbed copula such that summing along all other dimensions except dimension i still yields the marginal distribution $d\mu_i$.

A.2 Robustness to parameter variations

One possible further concern regarding our suggested method could be that it performs well only for the given parameterization of the Krusell and Smith model. For example, one question is whether it fares worse for calibrations that lead to more agents being borrowing constrained. To systematically evaluate this, we consider variations in model parameters as displayed in Table 9 and consider all possible parameter combinations.

Table 10 reports the mean values over all combinations for the mean and maximum absolute errors in aggregate capital for the Den Haan (2010a) statistics. Irrespective of the actual calibration, the method fares well with errors of the same order of magnitude as the Krusell and Smith method and the maximum error not exceeding 0.21%. When inspecting the error terms, we find that variations in the parameters that determine the steady state have virtually no impact on the quality of the approximation. The persistence and most importantly the standard deviation of shocks has a large impact

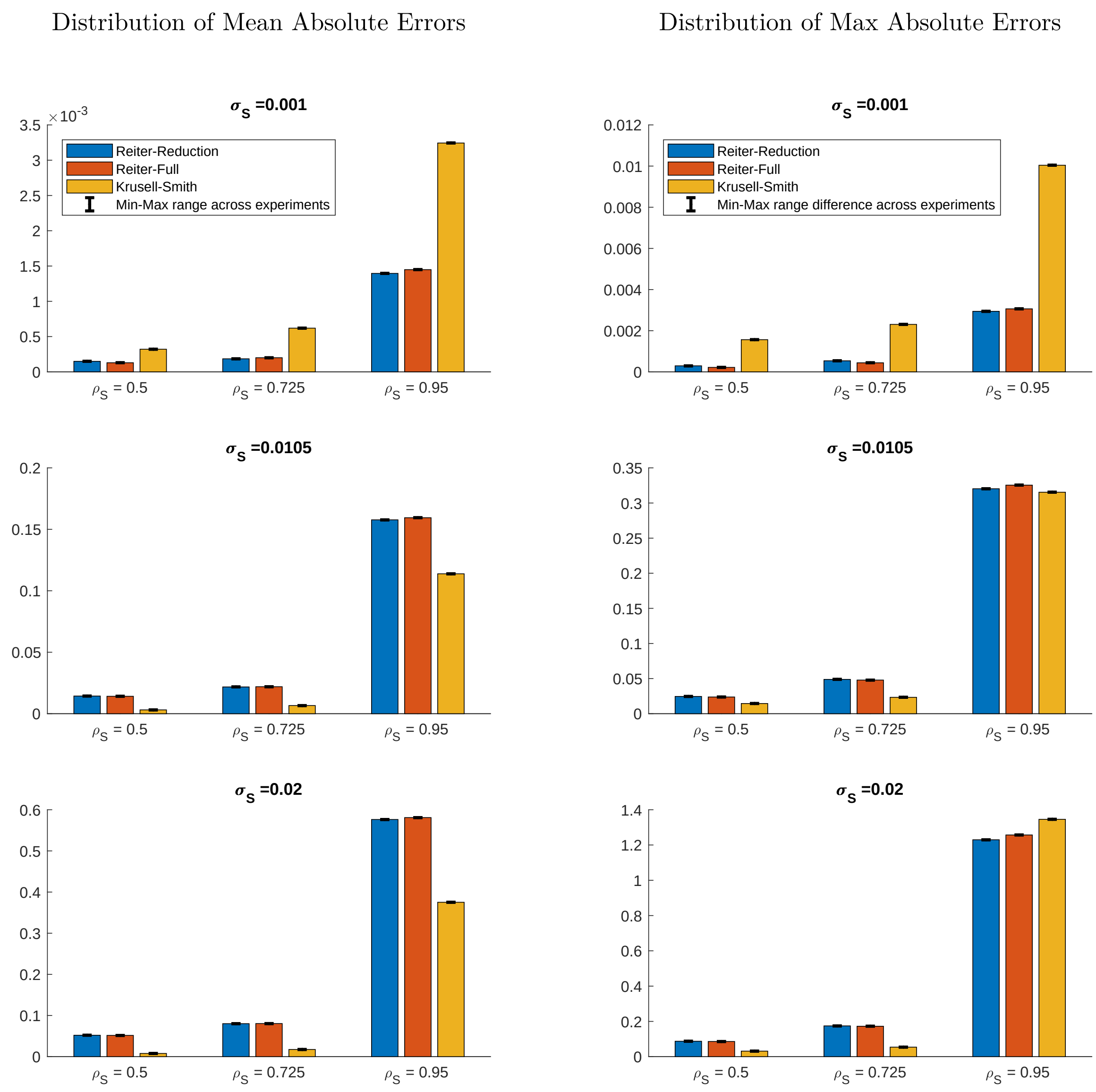
Table 9: Variations in model parameters

Relevant for steady state			
Parameter		Lower bound	Upper bound
β	Discount factor	0.95	0.99
ξ	Risk aversion	1	4
γ	Inv Frisch	0.5	2
ρ_H	Persistence (idiosyncratic)	0.7	0.95
σ_H	Variance (idiosyncratic)	0.05	0.4
α	Labor share	0.5	0.75
Only relevant for aggregate dynamics			
ρ_S	Persistence (aggregate)	0.5	0.95
σ_S	Variance (aggregate)	0.001	0.02

Notes: We use the lower and upper bound for each parameter to construct a linear spaced grid for this parameter. We then solve the model for all parameter combinations using 3 points for each grid, i.e. 6561 times.

on the quality of the approximation, see Figure 8. The figure displays the average mean- and max den Haan errors across for the nine (ρ_S, σ_S) combinations, we consider. Error bands for the largest and smallest den Haan errors across all parameters for a given (ρ_S, σ_S)-combination are also displayed but are so tight they can hardly be seen. What can be inferred from the graph is that all approximations become substantially worse when there is more aggregate uncertainty. The parameters that describe the household problem and the steady state have hardly any influence on the approximation quality and all methods fare substantially worse the larger is the variance of aggregate shocks, but also errors across methods are always in the same order of magnitude for a given size of shocks.

Figure 8: Distribution of Den Haan errors



Notes: Bars show average errors (in %) for combinations of steady state relevant parameters in Table 9, we solve the KS model by (1) the Reiter method with our proposed state-space reduction, (2) the original Reiter method without state-space reduction, (3) the original Krusell & Smith algorithm and report here mean and maximum absolute differences in percent between the simulation of the linearized solutions of the model and simulations in which we solve for the intra-temporal equilibrium prices in every period and track the full histogram over time for $t = \{1, ..., 1000\}$; see Den Haan (2010a). Error bands for maximal and minimal den Haan erorrs are displayed, too, but the min-max range is three orders of magnitude smaller than the average.

Table 10: Mean Den Haan errors

	Mean absolute error (in %) for capital K_t		
	Reiter-Reduction	Reiter-Full	K-S
Mean	0.1005	0.1012	0.0587
Max	0.2099	0.2129	0.1998

Notes: For all parameter combinations in Table 9, we solve the KS model by (1) the Reiter method with our proposed state-space reduction, (2) the original Reiter method without state-space reduction, and (3) the original Krusell & Smith algorithm and report here the *average* (over all parameter combinations) differences in percent between the simulation of the linearized solutions of the model and simulations in which we solve for the intra-temporal equilibrium prices in every period and track the full histogram over time for $t = \{1, ..., 1000\}$; see Den Haan (2010a).

A.3 Exploiting the structure of the problem to reduce the number of derivatives to be calculated

Our example code calculates all derivatives numerically without exploiting any of the models' structure. However, this leaves room to optimize calculations. This is particularly important for second-order derivatives.

First, we observe that the Fokker-Plank equation, the law of motion for distributions, is linear in the $d\mu$ terms and unaffected by current value functions (which are decision irrelevant). Second, we observe that the distribution terms $d\mu$ do not enter the Bellman equation as long as the set of controls (prices) is sufficiently rich and includes all individually decision relevant moments of distribution (typically only the means).

This allows to write the Jacobian of F in a convenient fashion which also reduces strongly the number of non-zero second-order derivatives. For this purpose, we reorder arguments of F and partition equations such that the "idiosyncratic" arguments and equations come first:

$$F\left(d\mu_t, \nu_t, S_t, P_t, d\mu_{t+1}, \nu_{t+1}, S_{t+1}, P_{t+1}, \varepsilon_{t+1}\right) = \begin{bmatrix} F^i\left(\cdot\right) \\ F^A\left(\cdot\right) \end{bmatrix} \tag{25}$$

$$F^i\left(\cdot\right) = \begin{bmatrix} d\mu_{t+1} - d\mu_t \Pi_{h_t} \\ \nu_t - \left(u_{h_t^d} + \beta \Pi_{h_t} \nu_{t+1}\right) \end{bmatrix} \tag{26}$$

$$F^A\left(\cdot\right) = \begin{bmatrix} X_{t+1} - H^X(X_t, D_t) + \varepsilon_{t+1} \\ D_{t+1} - H^D(X_t, D_t, d\mu_t) \\ \Phi_t(h_t^d, d\mu_t) \\ \varepsilon_{t+1} \end{bmatrix} \tag{27}$$

s.t.

$$h_t^d(s) = \arg \max_{d' \in \Gamma(x,d;P_t)} u(x,d,d') + \beta \mathbb{E} \nu_{t+1}(x', d'). \tag{28}$$

We can then write the Jacobian matrices A and B as

$$A = \begin{bmatrix} I & \partial_{\nu_{t+1}}(d\mu_t \Pi_{h_t}) & 0 & 0 \\ 0 & \partial_{\nu_{t+1}}\left(u_{h_t^d} + \beta \Pi_{h_t} \nu_{t+1}\right) & 0 & 0 \\ 0 & \begin{bmatrix} 0 \\ \partial_{\nu_{t+1}} \Phi(\cdot) \end{bmatrix} & \partial_{S_{t+1}} F^A(\cdot) & \partial_{P_{t+1}} F^A(\cdot) \end{bmatrix} \tag{29}$$

$$B = \begin{bmatrix} \Pi_{h_t} & 0 & \partial_{S_t}(d\mu_t \Pi_{h_t}) & \partial_{P_t}(d\mu_t \Pi_{h_t}) \\ 0 & I & \partial_{S_t}\left(u_{h_t^d} + \beta \Pi_{h_t} \nu_{t+1}\right) & \partial_{P_t}\left(u_{h_t^d} + \beta \Pi_{h_t} \nu_{t+1}\right) \\ \partial_{d\mu_t} F^A(\cdot) & 0 & \partial_{S_t} F^A(\cdot) & \partial_{P_t} F^A(\cdot) \end{bmatrix} \tag{30}$$

Here, we make use of the fact that future prices and states affect the policies only through future continuation values, that time-t value functions only affect the Bellman equation itself but are irrelevant for choices, and that the only effect of the current and future distributions is on the law of motion for distributions and on market clearing. All this yields a large number of (cross-)derivatives that are known to be zero.

What is more, we observe that the second-order derivatives of the idiosyncratic part F^i with respect to the distribution is zero as the Fokker-Planck equation is a linear equation in the distribution. Similarly, the second-order derivative with respect to the current value function is null, etc.

Once all derivatives are calculated, higher-order solutions require to solve a system of linear equations. Levintal (2017) shows how to write down higher-order derivatives in a compact way using matrix forms and provides code to efficiently solve large linear systems, which we use for our second-order solution.

B Calibrations

Table 11: Parameters of the Krusell & Smith model

Parameter	Value	Description	Source
Households			
β	0.99	Discount factor	Den Haan et al. (2010)
ξ	1	Relative risk aversion	Den Haan et al. (2010)
Production			
α	64%	Share of labor	Den Haan et al. (2010)
δ	2.5%	Depreciation rate	Den Haan et al. (2010)
ρ_Z	0.75	Persistence of productivity	Den Haan et al. (2010)
σ_Z	0.07	STD of innovations	Den Haan et al. (2010)

Notes: All values are reported for the quarterly frequency of the model. Idiosyncratic productivity follows the same two state Markov chain as in Den Haan et al. (2010).

Table 12: Parameters of the two-asset HANK model for Table 5

Parameter	Value	Description	Source
Households			
β	0.99	Discount factor	Den Haan et al. (2010)
ν	6.5%	Participation frequency	Luetticke (2018)
ξ	1	Relative risk aversion	Den Haan et al. (2010)
γ	1	Inv. Frisch elasticity	Standard value
$\overline{R}$	12.5%	Borrowing penalty	Bayer et al. (2019)
ρ_h	0.9	Persistence of productivity	Den Haan et al. (2010)
σ_h	0.25	STD of innovations	Den Haan et al. (2010)
ζ	0.0005	Prob. to become entrepreneur	Bayer et al. (2019)
ι	0.0625	Prob. to become worker	Guvenen et al. (2014)
Intermediate Goods			
α	67%	Share of labor	Den Haan et al. (2010)
δ	2.5%	Depreciation rate	Den Haan et al. (2010)
ρ_Z	0.75	Persistence of productivity	Den Haan et al. (2010)
σ_Z	0.07	STD of innovations	Den Haan et al. (2010)
Final Goods			
κ	∞	Price stickiness	0 quarters
η	20	Elasticity of substitution	5% markup
Capital Goods			
ϕ	0	Capital adjustment costs	Den Haan et al. (2010)
Fiscal Policy			
τ	0.3	Tax rate	$G/Y = 20\%$
ρ_B	0.5	Autocorrelation of debt	
γ_π	1.5	Reaction to inflation	Bayer et al. (2019)
γ_T	0.5	Reaction to taxes	Bayer et al. (2019)
Monetary Policy			
Π	1.0	Inflation	0% p.a.
R^B	1.0	Nominal interest rate	0% p.a.
θ_π	2.0	Reaction to inflation	Standard value
ρ_R	0.8	Interest rate smoothing	Standard value

Notes: All values are reported for the quarterly frequency of the model.

Table 13: Parameters of the two-asset HANK model for Figure 7

Parameter	Value	Description	Source
Households			
β	0.98	Discount factor	Bayer et al. (2019)
ν	6.5%	Participation frequency	Luetticke (2018)
ξ	4	Relative risk aversion	Bayer et al. (2019)
γ	1	Inv. Frisch elasticity	Bayer et al. (2019)
$\overline{R}$	11%	Borrowing penalty	Bayer et al. (2019)
ρ_h	0.98	Persistence of productivity	Bayer et al. (2019)
σ_h	0.06	STD of innovations	Bayer et al. (2019)
ζ	0.0005	Prob. to become entrepreneur	Bayer et al. (2019)
ι	0.0625	Prob. to become worker	Guvenen et al. (2014)
Intermediate Goods			
α	70%	Share of labor	Income share labor of 66%
δ	1.35%	Depreciation rate	NIPA: Fixed assets
Final Goods			
κ	0.09	Price stickiness	4 quarters
η	20	Elasticity of substitution	5% markup
Capital Goods			
ϕ	11.4	Capital adjustment costs	Bayer et al. (2019)
Fiscal Policy			
τ	0.3	Tax rate	$G/Y = 20\%$
ρ_B	0.5	Autocorrelation of debt	
γ_π	1.5	Reaction to inflation	Bayer et al. (2019)
$\gamma_\mathcal{T}$	0.50	Reaction to taxes	Bayer et al. (2019)
Monetary Policy			
Π	1	Inflation	0% p.a.
R^B	1.0062	Nominal interest rate	2.5% p.a.
θ_π	2.0	Reaction to inflation	Standard value
ρ_R	0.8	Interest rate smoothing	Standard value
Aggregate Shocks			
σ_Z	0.007	Standard deviation	TFP shock
ρ_Z	0.95	Persistence	TFP shock
σ_{R^B}	0.001	Standard deviation	Monetary shock
ρ_{R^B}	0.5	Persistence	Monetary shock
σ_s	0.84	Standard deviation	Uncertainty shock
ρ_s	0.54	Persistence	Uncertainty shock

Notes: All values are reported for the quarterly frequency of the model.