

Appendix: For Review and Online Publication Only

A Data

Income Categories: We define disposable income as income before tax minus reported federal, state and local income taxes payments, property tax not reported elsewhere and other tax (net of tax refunds), deductions for social security and pension plans. Household income before tax includes wages and salaries, net business income, net farm income, rents income, dividend income, interest income, pension income, social security and railroad retirement income, supplemental security income, unemployment compensation, workers' compensation and veterans benefits, welfare received, scholarship, food stamps, contributions received from others with alimony/child support, meals received as pay, rent received as pay, and lump sum receipts and lump sum child support payment.

Table A.I: Average Monthly Income and Consumption Expenditures - Sample

	Disp Income	Total Outlays	ND+Memorables	ND	Memorables	Durables
mean	1972.32	1484.61	1193.87	905.54	288.33	290.74

Table A.II: Average Demographic Characteristics - Sample

	Age of Head	Male Head	White Head	Married	High School Above	Family Size
mean	41.14	0.65	0.84	0.59	0.85	2.87

Table A.IV: Consumption Expenditure Shares (Quarterly Frequency)

	Outlays (%)	(ND+MG) (%)	Strictly (ND+MG) (%)
Outlays	100.0		
Durables	10.9		
ND+MG	89.1	100.0	
Memorables	18.6	21.2	
Nondurables	70.5	78.8	
Strictly (ND+MG)	77.0	86.4	100.0
Strictly Memorables	12.9	14.6	17.1
Strictly Nondurables	64.1	71.7	82.9

Table A.V: Consumption Expenditure Volatility (Quarterly Frequency)

	Had Zeros	Had Spikes	Ave Vol.
Outlays	0.000	0.274	0.321
Durables	0.419	0.911	1.184
ND+MG	0.000	0.084	0.208
Memorables	0.028	0.554	0.515
Nondurables	0.000	0.080	0.199
Strictly (ND+MG)	0.000	0.088	0.206
Strictly Memorables	0.053	0.609	0.569
Strictly Nondurables	0.000	0.078	0.195

A.1 Expenditure Spikes and Inactivity for Selected Detailed Goods Categories

In this section, we document the frequency of expenditure spikes and zero purchases for selected memorable goods categories: total expenditure on trips and vacations, clothes and shoes, jewelry and watches. We also report the expenditure spikes and zero purchase patterns for two durable goods categories, new and used vehicles (net outlay), and tires, tubes, accessories and other parts, as a comparison with memorable goods. Figure 4 shows that most households have at least one consumption expenditure spike within a 12 month period for these selected memorable and durable goods, and the expenditure on these goods tends to be quite lumpy. From Figure 5 we observe that indeed, memorable goods, as well as durable goods, display infrequent monthly expenditures.

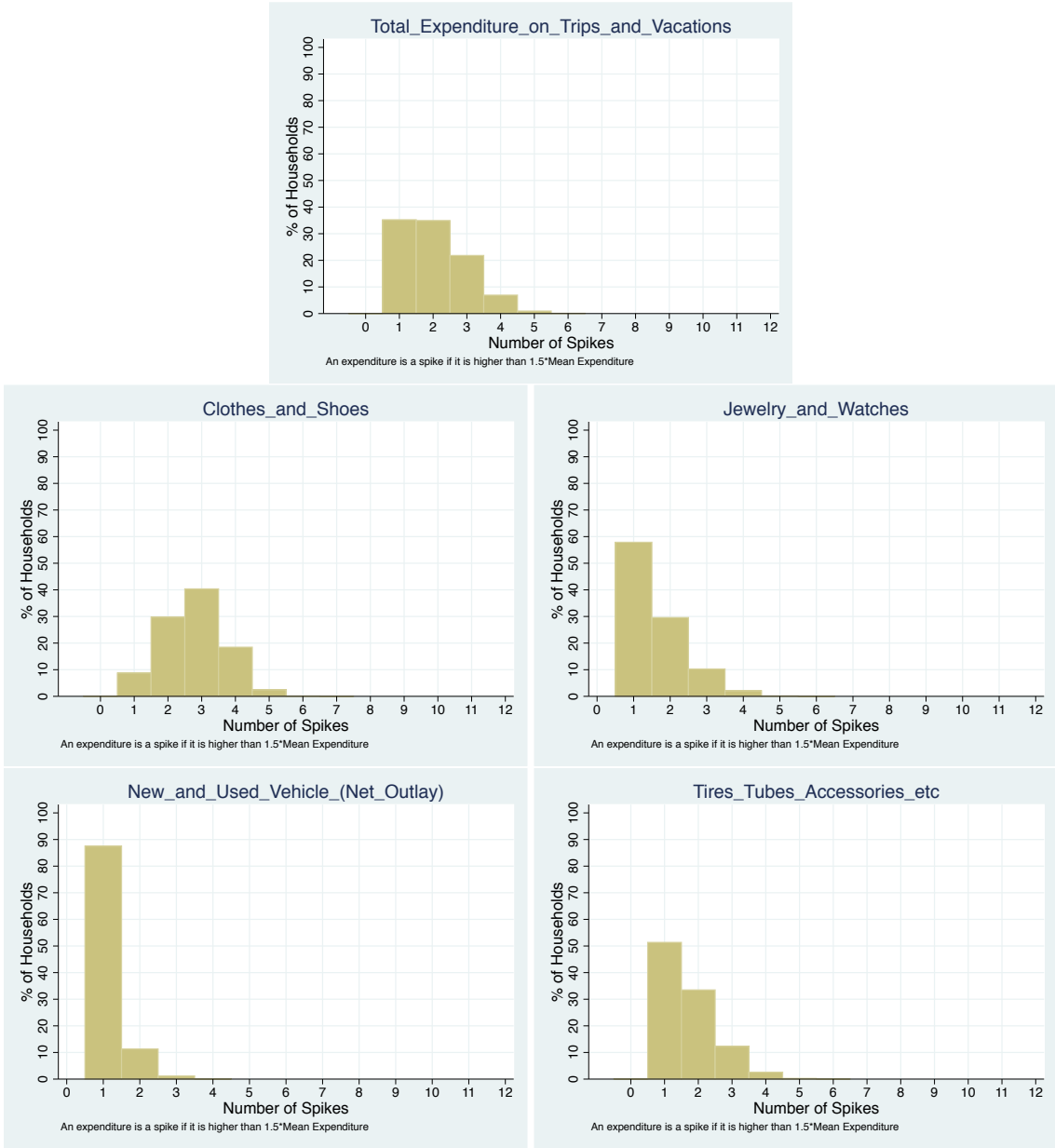


Figure 4: Number of Months with Expenditure Spikes ($\kappa = 1.5$)

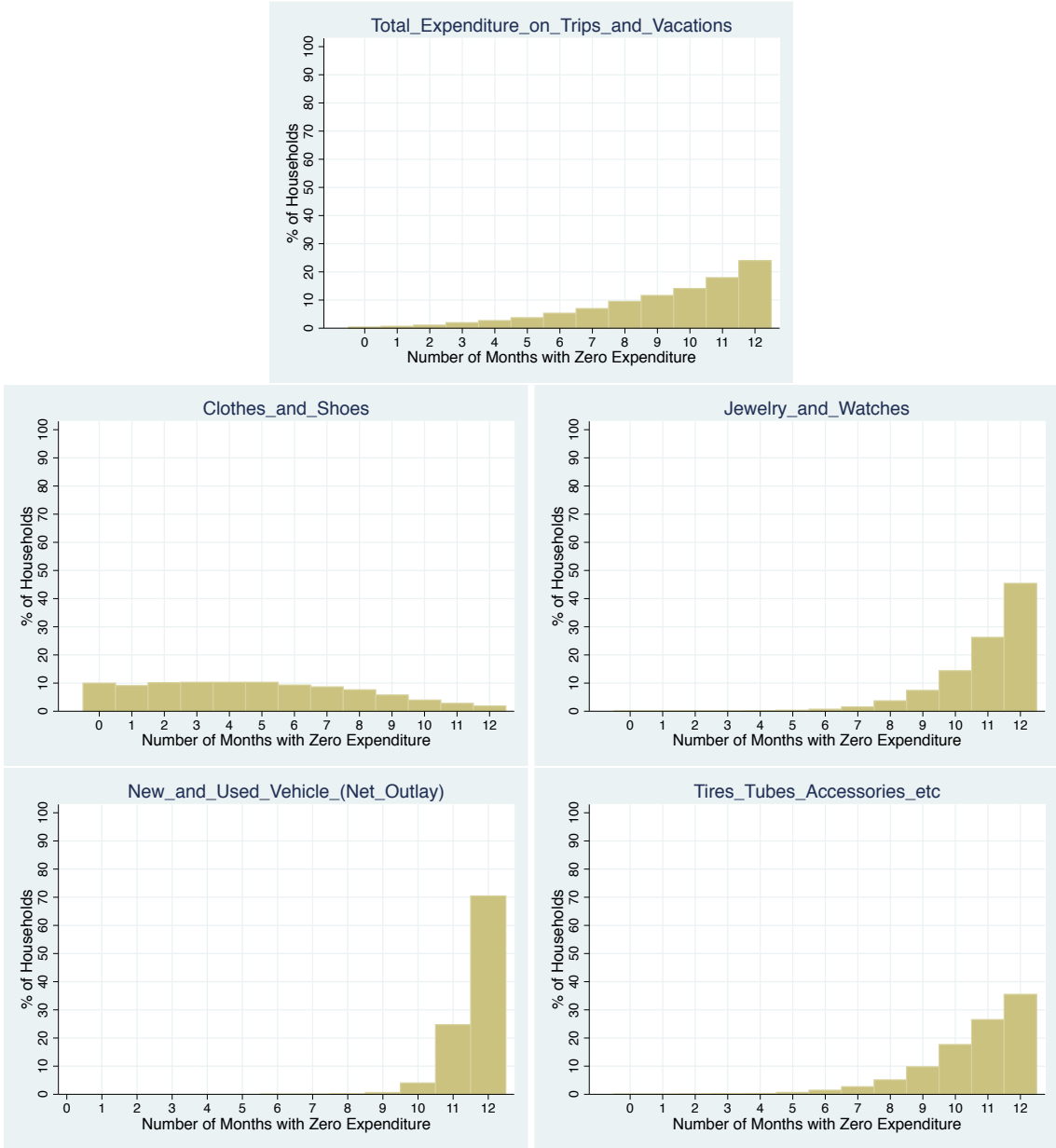


Figure 5: Number of Months with Zero Purchases

B Quantitative Model Derivations

Derivation of Euler Equations. The household's maximization problem is given by

$$\begin{aligned}
 V(M, N, S, z) &= \max_{C_m, S'} \{U(C_n, C_m, M) + \beta \mathbb{E}V(M', N', S', z')|z\} \\
 &\quad s.t. \\
 C_n &= Y + (1+r)S - C_m - S' \\
 M' &= (1 - \delta_m)M + \max\{C_m - N, 0\} \\
 N' &= (1 - \rho)N + \rho C_m \\
 S' &\geq 0 \\
 \ln Y &= \bar{y} + z \\
 z' &= \rho_z z + \varepsilon.
 \end{aligned}$$

We could rewrite the household's maximization problem as

$$\begin{aligned}
 V(M, N, S, z) &= \max_{N', S'} \{U(C_n, (N' - (1 - \rho)N)/\rho, M) + \beta \mathbb{E}V(M', N', S', z')|z\} \\
 &\quad s.t. \\
 C_n &= Y + (1+r)S - S' - \frac{1}{\rho}(N' - (1 - \rho)N) \\
 M' &= (1 - \delta_m)M + \frac{1}{\rho} \max\{N' - N, 0\} \\
 S' &\geq 0 \\
 \ln Y &= \bar{y} + z \\
 z' &= \rho_z z + \varepsilon.
 \end{aligned}$$

The first order conditions imply that the following two equations must hold at optimum,

$$\begin{aligned}\frac{\partial U}{\partial C_n}(C_n, C_m, M) &= \beta \mathbb{E} \frac{\partial V}{\partial S}(M', N', S', z') + \lambda_{S'} \\ \frac{\partial U}{\partial C_n}(C_n, C_m, M) - \frac{\partial U}{\partial C_m}(C_n, C_m, M) &= \mathbf{1}_{C_m > N} \cdot \beta \mathbb{E} \frac{\partial V}{\partial M}(M', N', S', z') + \rho \beta \mathbb{E} \frac{\partial V}{\partial N}(M', N', S', z')\end{aligned}$$

where $C_n = Y + (1+r)S - C_m - S'$, $\lambda_{S'}$ is the Lagrange multiplier associated with the borrowing constraint $S' \geq 0$, and $\mathbf{1}_{C_m > N}$ is an indicator function that equals 1 if and only if $C_m > N$.

The envelope theorem implies that the following conditions hold at the optimum,

$$\begin{aligned}\frac{\partial V}{\partial M}(M, N, S, z) &= \frac{\partial U}{\partial M}(C_n, C_m, M) + (1 - \delta_m) \beta \mathbb{E} \frac{\partial V}{\partial M}(M', N', S', z') \\ \frac{\partial V}{\partial N}(M, N, S, z) &= \frac{1 - \rho}{\rho} \frac{\partial U}{\partial C_n}(C_n, C_m, M) - \frac{1 - \rho}{\rho} \frac{\partial U}{\partial C_m}(C_n, C_m, M) - \mathbf{1}_{C_m > N} \cdot \beta \mathbb{E} \frac{\partial V}{\partial M}(M', N', S', z') \\ \frac{\partial V}{\partial S}(M, N, S, z) &= (1+r) \frac{\partial U}{\partial C_n}(C_n, C_m, M).\end{aligned}$$

The Euler equation for the optimal consumption path of nondurable goods C_n is standard,

$$\frac{\partial U}{\partial C_n}(C_n, C_m, M) - (1+r) \beta \mathbb{E} \frac{\partial U}{\partial C_n}(C'_n, C'_m, M') = \lambda_{S'}$$

where $\lambda_{S'}$ is the Lagrange multiplier associated with the borrowing constraint $S' \geq 0$.

Under our utility specification, the Euler equation of $C_{n,t}$ is given by the following equation

$$C_{n,t}^{-\gamma} - (1+r) \beta \mathbb{E}_t C_{n,t+1}^{-\gamma} = \frac{\lambda_{S_{t+1}}}{\xi}.$$

The optimal consumption path of memorable goods C_m rely on not only the borrowing constraint and the interest rate but also the memory stock M and the past experience level of memorable

goods consumption N ,

$$\begin{aligned}
& \frac{\partial U}{\partial C_n}(C_n, C_m, M) - \frac{\partial U}{\partial C_m}(C_n, C_m, M) \\
&= (1 - \rho)\beta \mathbb{E}\left(\frac{\partial U}{\partial C_n}(C'_n, C'_m, M') - \frac{\partial U}{\partial C_m}(C'_n, C'_m, M')\right) \\
&\quad + \mathbf{1}_{C_m > N} \cdot \beta \mathbb{E}\frac{\partial V}{\partial M}(M', N', S', z') - \rho\beta^2 \mathbb{E}(\mathbf{1}_{C'_m > N'} \cdot \mathbb{E}\frac{\partial V}{\partial M}(M'', N'', S'', z'')) \\
&= (1 - \rho)\beta \mathbb{E}\left(\frac{\partial U}{\partial C_n}(C'_n, C'_m, M') - \frac{\partial U}{\partial C_m}(C'_n, C'_m, M')\right) \\
&\quad + \mathbf{1}_{C_m > N} \cdot \beta \mathbb{E}\frac{\partial U}{\partial M}(C'_n, C'_m, M') \\
&\quad + \mathbf{1}_{C_m > N} \cdot \beta^2(1 - \delta_m) \mathbb{E}\frac{\partial V}{\partial M}(M'', N'', S'', z'') - \rho\beta^2 \mathbb{E}(\mathbf{1}_{C'_m > N'} \cdot \mathbb{E}\frac{\partial V}{\partial M}(M'', N'', S'', z'')).
\end{aligned}$$

Under our current utility specification, the above equation can be rewritten as

$$\begin{aligned}
C_{n,t}^{-\gamma} - \frac{1 - \xi}{\xi} K_t^{-\gamma} &= (1 - \rho)\beta \mathbb{E}(C_{n,t+1}^{-\gamma} - \frac{1 - \xi}{\xi} K_{t+1}^{-\gamma}) + \mathbf{1}_{C_m > N} \cdot \beta \zeta \frac{1 - \xi}{\xi} \mathbb{E}K_{t+1}^{-\gamma} \\
&\quad + \frac{1}{\xi} \mathbf{1}_{C_m > N} \cdot \beta^2(1 - \delta_m) \mathbb{E}\frac{\partial V}{\partial M}(M_{t+2}, N_{t+2}, S_{t+2}, z_{t+2}) \\
&\quad - \frac{1}{\xi} \rho\beta^2 \mathbb{E}(\mathbf{1}_{C_{m,t+1} > N_{t+1}} \cdot \frac{\partial V}{\partial M}(M_{t+2}, N_{t+2}, S_{t+2}, z_{t+2}))
\end{aligned}$$

where $K_t = C_{m,t} + \zeta M_t$. ■

Derivation of Euler Equations. Define

$$\begin{aligned}
\lambda_{n,t} &= \frac{\lambda_{S_{t+1}}}{\xi(1+r)\beta \mathbb{E}_t C_{n,t+1}^{-\gamma}} \\
\lambda_{m,t} &= \frac{\left((1 - \frac{(1-\rho)}{1+r})\xi C_{n,t}^{-\gamma} - \xi \frac{(1-\rho)}{1+r} \lambda_{S_{t+1}} - \mathbf{1}_{C_m > N_t} \cdot \beta \mathbb{E}_t \frac{\partial V}{\partial M}(M_{t+1}, N_{t+1}, S_{t+1}, z_{t+1}) \right)}{(1 - \xi)(1 - \rho)\beta \mathbb{E}_t ((C_{m,t+1} + \zeta M_{t+1})^{-\gamma})} \\
&\quad + \rho\beta^2 \mathbb{E}_t (\mathbf{1}_{C_{m,t+1} > N_{t+1}} \cdot \mathbb{E} \frac{\partial V}{\partial M}(M_{t+2}, N_{t+2}, S_{t+2}, z_{t+2}))
\end{aligned}$$

Rational expectations implies that at optimum the following equation must be true⁴¹:

$$\frac{(1+r)\beta C_{n,t+1}^{-\gamma}}{C_{n,t}^{-\gamma}}(1+\lambda_{n,t}) = 1 + e_{n,t+1} \quad (29)$$

$$\frac{(1-\rho)\beta[(C_{m,t+1} + \zeta M_{t+1})^{-\gamma}]}{(C_{m,t} + \zeta M_t)^{-\gamma}}(1+\lambda_{m,t}) = 1 + e_{m,t+1} \quad (30)$$

where $e_{n,t+1}$ and $e_{m,t+1}$ can be interpreted as the expectation error, and by construction $e_{n,t+1}$ and $e_{m,t+1}$ are uncorrelated with information known at time t . Taking logs on both side and taking a linear approximation⁴² of equation 29, we obtain the linearized Euler equation for nondurable consumption:

$$C_{n,t+1} - C_{n,t} = \frac{1}{\tilde{\gamma}} [\log((1+r)\beta) + \log(1+\lambda_{n,t}) - \log(1+e_{n,t+1})]. \quad (31)$$

Note that when the borrowing constraint is not binding at period t ($\lambda_{S_{t+1}} = 0$) $\lambda_{n,t} = 0$.

Doing the same with equation 30 yields

$$C_{m,t+1} - C_{m,t} = \frac{1}{\hat{\gamma}} [\log((1-\rho)\beta) + \log(1+\lambda_{m,t}) - \log(1+e_{n,t+1})] - \zeta(M_{t+1} - M_t)$$

and plugging in the law of motion for M_{t+1} delivers the linearized Euler equation for memorable consumption expenditures:

$$C_{m,t+1} - C_{m,t} = \frac{1}{\hat{\gamma}} [\log((1-\rho)\beta) + \log(1+\lambda_{m,t}) - \log(1+e_{n,t+1})] - \zeta(-\delta_m M_t + \max\{C_{m,t} - N_t, 0\}) \quad (32)$$

In these equations the constants $\tilde{\gamma}$, $\hat{\gamma}$ are products of the risk aversion coefficient γ and approximation constants. ■

⁴¹ See Parker and Preston (2005) and Parker (1999) for similar analyses for nondurable goods expenditure.

⁴² The linear approximation used here is $\log y_{t+1} - \log y_t = (y_{t+1} - y_t)/\bar{y}$ for some \bar{y} .

C Model Solution Algorithm

The model solution algorithm is as follows:

Step 1. Guess an initial value of value function $V^{(0)}$ at each grid point of the state space, use OLS regression to calculate the Smolyak coefficients associated with value function $V^{(0)}$.

Step 2. At each state space grid point, value function at the i -th iteration, $V^{(i)}$, is maximized by searching memorable goods consumption C_m over a discrete grid

$$V^{(i)}(M, N, S, z) = \max_{C_m \in \text{Grid of } C_m} \{W^{(i)}(M, N, S, z, C_m)\}$$

where $W^{(i)}(M, N, S, z, C_m)$ is the value function associated with memorable goods consumption C_m for given state space variables (M, N, S, z) , i.e.,

$$W^{(i)}(M, N, S, z, C_m) = \max_{S'} \left\{ U(C_n, C_m, M) + \beta \mathbb{E}[V^{(i-1)}(M', N', S', z') | z] \right\}.$$

The solution of optimal savings S'^* associated with memorable goods consumption C_m is characterized by the following equation

$$-\frac{\partial U(Y + (1+r)S - C_m - S'^*, C_m, M)}{\partial C_n} + \beta \frac{\partial \mathbb{E}[V^{(i-1)}(M', N', S'^*, z') | z]}{\partial S'} = 0$$

and $S'^* = 0$ if $-\frac{\partial U(Y + (1+r)S - C_m, C_m, M)}{\partial C_n} + \beta \frac{\partial \mathbb{E}[V^{(i-1)}(M', N', 0, z') | z]}{\partial S'} \leq 0$.

For (M', N', S', z') outside the state space grid, the value of value function $V^{(i-1)}(M', N', S', z')$ is calculated via interpolation using Smolyak coefficients. Furthermore, $\mathbb{E}[V^{(i-1)}(M', N', S'^*, z') | z]$ is calculated using quadratic rule numerical integration method.

Step 3. Update Smolyak coefficients associated with value function $V^{(i)}$.

Step 4. Repeat Step 2 to 3 until the value of value function at each state space grid point and associated Smolyak coefficients converge.

D Computation of Stationary Distribution

Our model predicts that there is a cross-sectional stationary distribution of state variables. There is no analytical solution to the household's consumption-savings problem, so we characterize the cross-sectional distribution of (M_t, N_t, S_t, z_t) numerically using Markov chain Monte Carlo (MCMC) simulation method. Specifically our procedure is as follows:

Step 1: At period $t = 0$, we randomly simulate state variables (M_0, N_0, S_0) for each household $h \in \{1, \dots, H\}$ from an arbitrary initial distribution $F^{(0)}(M, N, S)$, and draw z_0 from the distribution $N(0, \sigma^2/(1 - \rho^2))$ for each household.

Step 2: At period $t = 0$, for given state variables (M_t, N_t, S_t, z_t) , households optimally make their current memorable goods consumption $C_{m,t}^*$ and period $t + 1$ savings decisions S_{t+1}^* . Households' period $t + 1$ state variables M_{t+1}^* and N_{t+1}^* are updated according to Equations 20 and 11 respectively. Households' period $t + 1$ income shock z_{t+1} is randomly drawn according to the conditional distribution $N(\rho_z z_t, \sigma^2)$. The updated state variables $(M_{t+1}^*, M_{t+1}^*, M_{t+1}^*)$ for H households yield the numerical distribution $F^{(1)}(M, N, S)$.

Step 3: Check if distribution $F^{(1)}(M, N, S)$ converges to $F^{(0)}(M, N, S)$ by checking whether the mean and variance of the state variable M, N, S are the same under these two distributions. If the distribution has not converged, repeat step 2 for $t = 2, \dots$

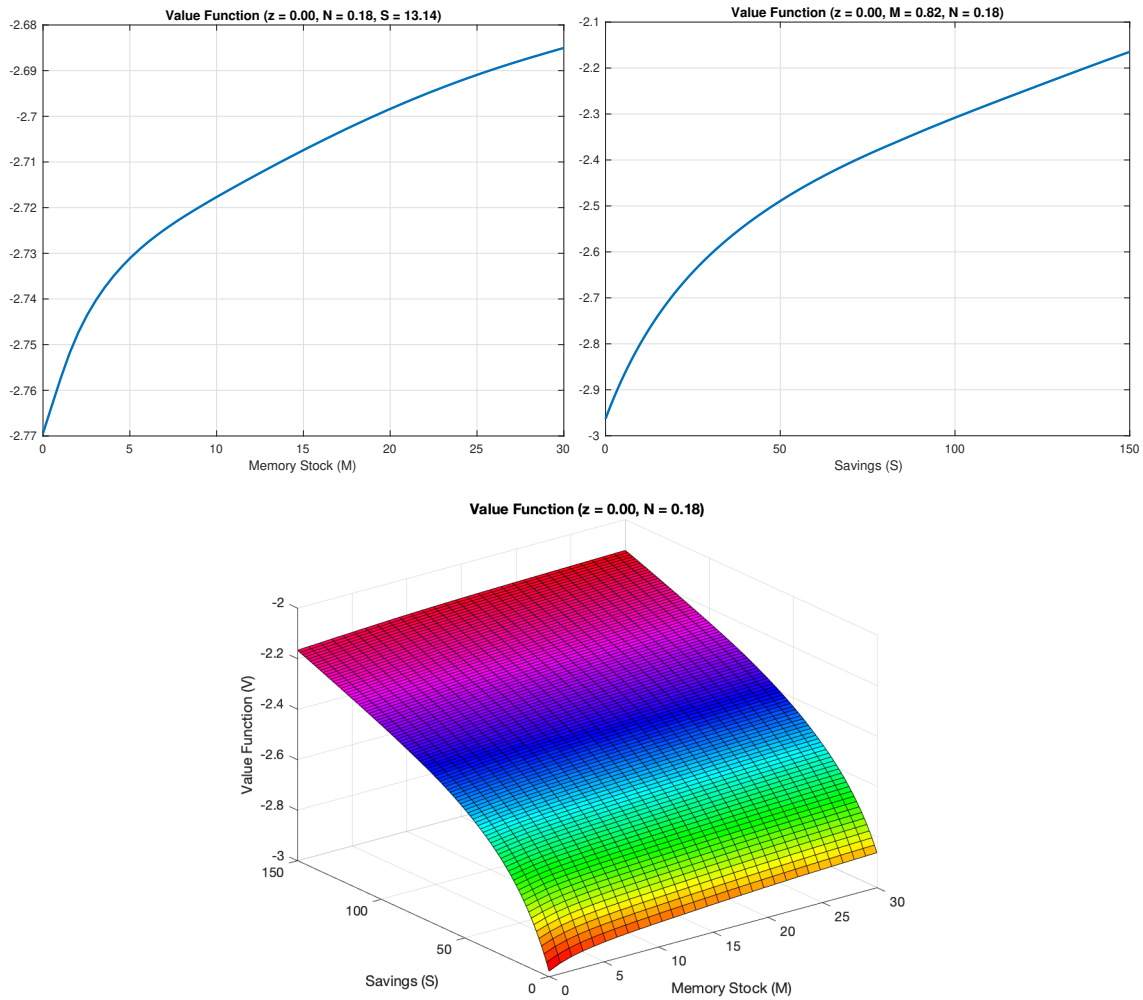


Figure 6: Value Function

E Welfare Cost Analysis

Derivation of Equation 25. Define as

$$\begin{aligned} \bar{V}(M, N, S; g) = & \xi \frac{[(1-g)\bar{C}_n(M, N, S)]^{1-\gamma}}{1-\gamma} + (1-\xi) \frac{((1-g)\bar{C}_m(M, N, S) + \zeta(1-g)M)^{1-\gamma}}{1-\gamma} \\ & + \beta \bar{V}(\bar{M}', \bar{N}'(M, N, S), \bar{S}'(M, N, S); g). \end{aligned}$$

Note that

$$\bar{V}(M, N, S; g) = (1-g)^{1-\gamma} \bar{V}(M, N, S; g=0) = (1-g)^{1-\gamma} \bar{V}(M, N, S). \quad (33)$$

As for \bar{V} , we can define $\bar{W}(S; g)$ by

$$\bar{W}(S; g) = \frac{((1-g)\bar{C}_n^W(S))^{1-\gamma}}{1-\gamma} + \beta \bar{W}(\bar{S}^{W'}(S); g).$$

Note that

$$\bar{W}(S; g) = (1-g)^{1-\gamma} \bar{W}(S; g=0) = (1-g)^{1-\gamma} \bar{W}(S). \quad (34)$$

$\bar{V}(M, N, S; g)$ is lifetime utility in the no-risk economy with memorable goods, but with non-durable and memorable consumption scaled up by a factor g at all future dates. The function $\bar{W}(S; g)$ has a similar interpretation.

For $\gamma = 1$, a similar calculation yields

$$\begin{aligned} \bar{V}(M, N, S; g) &= \frac{\log(1-g)}{1-\beta} + \bar{V}(M, N, S; g=0) = \frac{\log(1-g)}{1-\beta} + \bar{V}(M, N, S) \\ \bar{W}(S; g) &= \frac{\log(1-g)}{1-\beta} + \bar{W}(S; g=0) = \frac{\log(1-g)}{1-\beta} + \bar{W}(S). \end{aligned}$$

The welfare cost of consumption fluctuations for a household in state (M, N, S) is then defined (in

the model with and without memorable goods, respectively) as the solution to

$$\begin{aligned}\bar{V}(M, N, S; g(M, N, S)) &= V(M, N, S, z = 0) \\ \bar{W}(S; g^W(S)) &= W(S, z = 0)\end{aligned}$$

where setting $z = 0$ in the model with risk again assures that households have the same income today and same expected income from tomorrow on in both worlds. Solving for $g(M, N, S)$ and $g^W(S)$ gives, exploiting equations (33) and (34),

$$\begin{aligned}1 - g(M, N, S) &= \left[\frac{V(M, N, S, z = 0)}{\bar{V}(M, N, S)} \right]^{\frac{1}{1-\gamma}} \\ 1 - g^W(S) &= \left(\frac{W(S, z = 0)}{\bar{W}(S)} \right)^{\frac{1}{1-\gamma}}.\end{aligned}$$

■

F Revisiting an Excess Sensitivity Test of Consumption: Data and Sample Selection

To insure comparability with Souleles (1999) our empirical strategy, as well as crucial sample selection choices and variable definitions, follows his as much as possible. Our definition of non-durable and memorable goods is the same as in previous sections. As discussed in Section 3, our definition of nondurable and memorable goods combined is equivalent to Souleles (1999)'s non-durable goods (ND+MG), and our definition of strictly nondurable and strictly memorable goods combined equals Souleles (1999)'s definition of strictly nondurable goods (Strictly (ND+MG)).⁴³

The sample was selected in a way that closely follows the selection criteria provided in Souleles

⁴³The major components of strictly nondurables, defined in Souleles (1999), are food; household operations, including monthly utilities and small-scale rentals; apparel services and rentals; transportation fuel and services; personal services; and entertainment services and high-frequency fees. We further break down the above consumption groups into two consumption categories: strictly nondurable and strictly memorable goods by introducing memorable goods.

(1999).⁴⁴ The CEX asks about tax refunds twice, in a household's first and final interview. Each time what is recorded is the value of federal tax refunds received by the households in the 12 months before the interview month. Thus the refund variable in the CEX has a reference period of 12 months. About 80 percent of the refunds were mailed in March, April and May during the years 1980-1991,⁴⁵ and thus following Souleles (1999), we deflate refunds by the average of the monthly CPI for all items averaged over March, April, and May. All nominal variables were deflated to 1982-1984 dollars.

⁴⁴A household was dropped from the sample if there were multiple consumer units in the household, or if the household lived in student housing or the head of household was a farmer; a household quarter was dropped if the household lacked basic food expenditure for any month of the quarter, or if any food was received as pay in the quarter. A household quarter is dropped if the age of household head increased by more than one or decreased moving into next quarter. The sample was restricted to households with heads aged 24-64. Finally, a household is dropped if the income report is incomplete or any of the income or financial records are invalid. We thank Nick Souleles for sharing the data appendix of Souleles (1999).

⁴⁵Refer to Table 2 in Souleles (1999).