

Waiting for Affordable Housing in New York City*

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Abstract

We develop a new dynamic equilibrium model with heterogeneous households that captures the most important frictions that arise in housing rental markets and explains the political popularity of affordable housing policies. We estimate the model using data collected by the New York Housing Vacancy Survey in 2011. We find that there are significant adjustment costs in all markets as well as serious search frictions in the market for affordable housing. Moreover, there are large queuing frictions in the market for public housing. Having access to rent-stabilized housing increases household welfare by up to \$65,000. Increasing the supply of affordable housing by ten percent significantly improves the welfare of all renters in the city. Progressive taxation of higher-income households that live in public housing can also be welfare improving.

Keywords: Affordable Housing, Urban Housing Policies, Excess Demand, Housing Supply, Moving Costs, Rationing, Search Frictions, Queuing, Welfare Analysis, Political Economy of City Governments.

JEL classification: C33, C83, D45, D58, H72, R31.

1 Introduction

As the U.S. has shifted toward a knowledge-based economy, most large cities in the U.S. have experienced a surge in popularity. As a consequence, real estate prices and rental rates for housing have soared in many U.S. cities for the past two decades. These increases in the prices of housing can be explained by an inelastic supply of housing due to restrictive zoning laws combined with increasing demand of high-income households who prefer to live in attractive cities.¹ These economic trends have created a shortage of affordable housing in the most expensive housing markets such as Boston, Los Angeles, New York, San Francisco, and Seattle. Many progressive politicians have included the expansion of affordable housing policies in their platforms.

The political popularity of affordable policies in these cities is in stark contrast to long term trends in the supply of affordable housing. To increase the supply of housing, local governments often need to change zoning laws. Landowners and housing developers reap substantial windfall gains from rezoning. Politicians can redistribute part of these windfall gains from rezoning to renters by mandating that a certain amount of new housing is offered at affordable rates. The key insight here is that most affordable housing policies are not necessarily motivated by efficiency gains, but are driven by political conveniences. It is well-known that affordable housing policies create a mismatch and often reduce the incentives to increase housing supply. The political popularity of these policies simply reflect the wishes and needs of the majority of voters that tend to be low- and moderate-income renters.²

Despite the importance, prevalence, and political popularity of affordable housing policies in high-cost cities, there are few compelling dynamic models that allow us to quantify the benefits that arise from these policies to low- and moderate-income households.³ The objec-

¹See Gyourko, Mayer, and Sinai (2013) for a more detailed discussion of housing trends in large U.S. cities.

²In that sense, the current political popularity of affordable housing policies may just reflect the “tyranny-of-the-majority.”

³Low- and moderate-income households play a key role in the provision of many local goods and services in

tive of this paper is to fill this gap. We develop and estimate a new dynamic equilibrium model with heterogeneous households that is consistent with the observed market frictions, the presence of mobility or adjustment costs, the existence of long queues for public housing, and the need to search for a long time to obtain access to rent-stabilized housing. Our model, therefore, captures the existence of three different types of rental markets – public, regulated, and unregulated rental markets – and formalizes the dynamic incentives faced by households.

We assume that households can rent any amount of housing in the unregulated market given the prevailing market price per unit of housing services. However, moving is costly. This implies that a household faces some adjustment costs when moving to a new rental unit that better fits its needs. All households also have access to regulated or rent-stabilized housing. We assume that the rental price for rent-stabilized housing is significantly lower than the equivalent market price in the unregulated market. Since the demand for rent-stabilized units typically exceeds the supply, there are significant frictions in the rent-stabilized market. Finding a rent-stabilized apartment involves significant search efforts and luck. We capture these market frictions by endogenizing the probability that a household who is actively searching for rent-stabilized housing receives an offer to move into a stabilized unit. The search frictions and moving costs then create misallocations of housing units in rental markets.⁴

Low- and moderate-income households are also eligible for public housing assistance in our model if their incomes are below an exogenously determined threshold. The rent charged for public housing is a fixed percentage of household income. Hence, there is no price mechanism

an urban economy. Their presence is particularly essential with extreme-skill complementarity in the production function of large cities as discussed in detail by Eeckhout, Pinheiro, and Schmidheiny (2014).

⁴Our modeling approach is thus consistent with Glaeser and Luttmer (2003) who show that rent controls lead to misallocations in housing markets. Our paper is also related to search and matching models that have been applied to study owner-occupied housing markets. See, for example, Wheaton (1990), Krainer (2001), Albrecht, Anderson, Smith, and Vroman (2007), Piazzesi and Schneider (2009), Diaz and Jerez (2013), Anenberg and Bayer (2013), and Anenberg and Kung (2017). Owner-occupied housing is distinctly different from affordable and public housing that are the focus of this paper.

to ensure that public housing markets clear. Demand for public housing vastly exceeds the available supply in most affluent U.S. cities. Rationing is achieved by placing households on waitlists that allocate free units to households with the highest priority, i.e. households that have waited for public housing the longest.⁵ Moreover, the housing authority does not evict households after they have lost their eligibility for housing aid. Consequently, these policies imply that public housing is not necessarily allocated to the most deserving low-income households.

Our model significantly improves over previous studies that have been based on static models. Any model that tries to capture search or queuing behavior needs to be dynamic. The main challenges relate to modeling the degree of forward-looking behavior and the beliefs that households hold with respect to the probabilities of obtaining access to public or affordable housing. Public and affordable housing also offers partial insurance against negative income shocks. Upon receiving a positive income shock, a household may consider moving out from public housing or rent-regulated housing. However, households need to take into consideration that they may be hit by negative income shock in the future and that it will take a long time to get back into public or affordable housing. These insurance aspects are potentially important components of the welfare gains associated with public or affordable housing.

We define and characterize a stationary equilibrium with search and rationing. The length of the waitlist for public housing and the probability of finding a rent-stabilized unit are all endogenously determined in the equilibrium of our model. Low- and moderate-income households prefer to live in public housing due to the large rent subsidy, which implies a large increase in numeraire consumption, and the relatively high quality of these housing units.

⁵Geyer and Sieg (2013) consider a static model of public housing with myopic households and do not analyze rent-stabilization programs. Moreover, they focus on housing markets in Pittsburgh. Thakral (2015) considers a dynamic model of matching and introduces a multiple waitlist procedure. His analysis suggests that there are large potential welfare gains associated with this allocation mechanism. Halket and Nesheim (2017) also consider the problem of optimal allocation of public housing.

Rent-stabilized housing appeals to a large range of low-, moderate- and even high-income households due to the significant rental price discount relative to the unregulated market. Due to the existence of rationing in public housing and search frictions in rent-stabilized housing a fraction of low- and moderate-income households must also rent in the unregulated market in equilibrium. Our model can, therefore, explain the existence of long wait and search times and is consistent with the observed misallocations in public and rent-stabilized housing markets.

The parameters of our model can be identified based on the observed moments in the data. Our proof of identification is constructive and can be used to define a method of moments estimator. This estimator matches the sorting of households by income and family type among housing options and the average time spent in different housing markets. The estimator also matches the average rental payments for each housing type.⁶

Our empirical analysis focuses on New York City (NYC). While many cities in the U.S. and abroad face the challenge to provide an adequate supply of affordable housing, NYC has been at the center of the debate over the desirability of affordable housing policies. Studying the housing markets for low- and moderate-income households in NYC is promising for a variety of compelling reasons. First, NYC has the largest stock of rental apartments of all cities in the U.S. and is generally perceived to be one of the most expensive rental markets in the world. Second, New York City also has the largest stock of public housing units of all cities in the U.S. Finally, NYC is the only large city in the United States that has ever declared a housing emergency and has adopted strict rent-stabilization policies over an extended period of time.⁷ NYC, therefore, serves as a laboratory to explore the effectiveness and impact of a affordable housing policies.

⁶Our work is also related to the new literature on estimating dynamic models of houses and neighborhood choice, as discussed in Bayer, McMillan, Murphy, and Timmins (2016).

⁷Over one million households live in rent-regulated housing units in New York City. Many other large European and Asian cities also use strict rent control laws to provide affordable housing. An early analysis of the benefits and costs of public housing in New York City is given by Olsen and Barton (1983).

Our empirical analysis is based on the 2011 sample of the New York City Housing and Vacancy Survey (NYCHVS). This survey provides comprehensive data about household and housing characteristics. In particular, we observe household income and family status, the time that the household has spent in the housing unit, as well as a large number of structural characteristics of the housing unit that the household occupies. In addition, it allows us to classify households as living in public housing, rent-stabilized housing, or unregulated housing. We implement our estimator focusing on Manhattan since waitlists for public housing in NYC are operated at the borough level. The data show that approximately 10 percent of our sample of low- and moderate-income households lived in public housing communities in 2011. 57 percent of households lived in rent-stabilized units. The remaining 33 percent rented in the unregulated housing market. At the time of the survey, households spent, on average, 16 years in public housing, 9.5 years in regulated housing and only 4 years in unregulated housing. Not surprisingly, households in public housing are much poorer than households in rent-stabilized and unregulated housing.

Our model fits the sorting of households by income among the three housing options. It captures differences in rental prices as well as time spent in the housing units. Our model allows us to quantify the importance of search and queuing frictions, as well as adjustment costs. Our findings suggest that all three types of frictions are highly relevant to understand rental markets in NYC. We find that rental prices for stabilized housing are approximately 50 percent lower than prices in the unregulated market in Manhattan. This significant discount explains the popularity of rent-stabilized units. The probability of finding a rent-stabilized unit is approximately 11 percent per year for a relatively affordable, low-quality unit and 26 percent for a more expensive, high-quality unit. The average wait time for public housing is approximately 19 years.

Policymakers and politicians have struggled to find a response to the increasing demands of low- and moderate-income voters to increase the supply of affordable housing. Our model provides a compelling explanation of why these policies have been so popular at the ballot box

with the vast majority of urban renters. Our analysis shows that low-quality units of affordable housing are attractive for most low- and moderate-income households. These households gain up to \$60,000 from having access to stabilized units. For high-quality units, the results are even more striking. High-quality units are attractive for moderate- and high-income households and generate welfare gains of up to \$65,000.⁸ Given these large benefits associated with affordable housing, it is not surprising that rent-stabilization and affordable housing policies are popular, not only with low- and moderate-income households, but with the vast majority of all urban renters in NYC.

We then evaluate the impact of increasing the supply of affordable housing on the distribution of renters' welfare. We find that a ten percent increase in the supply of affordable improves welfare for all renters since the wait and search times decrease. The average welfare gains associated with a permanent 10 percent increase in the stock of affordable housing are approximately \$20,000. Of course, these findings do not imply that these policies are desirable or efficient since these policies impose large losses on landowners and housing developers. The magnitude of these losses are hard to assess.⁹ However, our findings provide a compelling explanation of why affordable housing policies are increasingly popular among urban renters and populist politicians.

Finally, we study the impact of policies that progressively tax households that live in public housing, but have incomes that exceed the eligibility threshold. Note that approximately 17 percent of all household living in public housing in Manhattan fall into that group. We show that we can design a revenue-neutral progressive tax system that can increase overall welfare by taxing ineligible high-income households at a substantially higher rate than eligible, low- and moderate-income households. Moreover, we can also use progressive taxation to reduce the average wait time for public housing and reallocate public housing to more deserving

⁸The gains associated with public housing are of a similar magnitude, approximately \$60,000. These findings are robust to a number of changes in the model specification.

⁹The early literature on rent control primarily focus on the misallocation in housing markets. See, for example, Olsen (1972), Suen (1989), and Gyourko and Linneman (1989).

low-income households.

The rest of the paper is organized as follows. Section 2 discusses affordable housing policies in NYC and our data. Section 3 provides a new dynamic model of affordable housing markets. Section 4 discusses identification and estimation of the parameters of our model. Section 5 presents our empirical findings. Section 6 reports the findings from our welfare analysis and considers alternative housing policies. Section 7 offers our conclusions.

2 Data

Our empirical analysis focuses on the rental housing markets of NYC. Housing markets have been heavily regulated in NYC since the 1930s. As of 2011, over one million units were rent-stabilized representing roughly 47 percent of the rental housing stock in NYC.¹⁰ Rent-stabilization generally applies to buildings of six or more units built between February 1, 1947, and December 31, 1973, and to those units that have exited from the rent-control program. Approximately 8 percent of the city’s stabilized units and nearly all stabilized units in buildings constructed after 1974 were voluntarily subjected to rent-stabilization by their owners in exchange for tax incentives from the city. Under the 421-a program, developers currently have to set aside 20 percent of new apartments for poor and working-class tenants to receive tax abatements lasting 35 years.¹¹

Involuntarily stabilized units, representing 92 percent of the stabilized stock, are regulated based on a “housing emergency” declared by the city in 1974 and renewed every three years since. Under New York State’s Rent-stabilization Law, the city may declare a housing emergency whenever the city’s rental vacancy rate drops below five percent. This law was most recently renewed in June 2015 and affects units with a maximum rent of \$2,700. Rent stabiliza-

¹⁰The stock of rent-regulated units includes a relatively small number of rent controlled units - approximately 38,000 units which are primarily older.

¹¹The de Blasio administration has been pushing to increase that fraction to 35 percent.

tion sets maximum rates for annual rent increases. It also entitles tenants to have their leases renewed. The rent guidelines board meets every year to determine how much the landlord can set future rents on the lease.

In addition, low- and moderate-income households may have access to public housing. Providing adequate housing and shelter for low- and moderate-income households has been a policy goal of most federal, state, and city administrations in the United States since the passage of the Public Housing Act of 1937. The New York City Housing Authority (NYCHA) provides public housing and administers Section 8 housing vouchers for low- and moderate-income residents throughout the five boroughs of New York City. Households whose incomes do not exceed 80% (50%) of median income are eligible for the public housing program (voucher program). In addition, income limits are functions of family size. For example, in 2011 the income limit for a single person household was \$45,850 (\$28,500) while it was \$65,450 (\$40,900) for a family of four.

Applications for public housing are assigned a priority code based upon information that includes employment status, income, family size, and quality of the previous residence provided. Households are then placed on the housing authority's preliminary waiting list for an eligibility interview. Households are required to update or renew their applications every two years if they have not been scheduled for an interview. Upon passing the interview and background checks, applicants are then placed on a (borough-wide) waiting list.

More than 403,000 New Yorkers reside in NYCHA's 177,666 public housing apartments across the city's five boroughs. Another 235,000 residents receive subsidized rental assistance in private homes through the NYCHA-administered Section 8 program. The NYCHA reported that 270,201 families were on the wait list for conventional public housing and 121,356 families on the wait list for Section 8. Little is known about the annual flows of waitlisted individuals into public housing. The NYT reported on July 23, 2013, that "the queue moves slowly. The apartments are so coveted that few leave them. Only 5,400 to 5,800 open up annually." As of December 10, 2009 NYCHA stopped processing any new Section 8 applications due to the

long wait list. As a consequence, there is almost no mobility in and out of Section 8 housing markets. We, therefore, treat Section 8 housing as a completely separate market and focus on public housing and rent-stabilized housing in this paper.

The empirical analysis is based on the New York City Housing Vacancy Survey (NYCHVS) in 2011. The main advantage of this data set is that it matches households with units. It contains detailed information about both household characteristics and housing characteristics.

Table 1: Descriptive Statistics

housing type	market share	rent	number of years	income	female head	kids	working family
Public	0.10	—	16.18	32,930	0.73	0.92	0.70
Regulated	0.57	1317	9.49	54,739	0.53	0.38	0.83
Unregulated	0.33	2640	3.85	71,045	0.54	0.17	0.87

Source: New York City Housing Vacancy Survey 2011.

A household is defined as working if the labor income share is higher than 50 percent of total income.

Regulated units include rent-stabilized units, HUD-regulated units, and Michell-Lama rental units.

We focus on affordable housing for low- and moderate-income households which imposes three sample restrictions. First, we drop households whose average incomes exceed 200% of median income level. This sample restriction is motivated by the fact that many high-income households are likely to own a condominium or house and, therefore, face a different choice set than low- and moderate-income households.¹² Second, we drop all low-income households that receive vouchers since that market has been closed for at least 6 years.¹³ Finally, we drop all households not living in Manhattan since waitlists are operated at the borough level rather than city-wide. These restrictions reduce our sample size to 1,666.¹⁴

¹²None of the key findings of this paper qualitatively or quantitatively depend on these choices.

¹³Galiani, Murphy, and Pantano (2015) estimate a model of neighborhood choice with vouchers.

¹⁴Descriptive statistics for the full sample that includes renters from all five boroughs of NYC are qualitatively similar. Details available upon request from the authors.

Table 1 provides some descriptive statistics of the Manhattan housing market for 2011. Table 1 shows that a large fraction of the rental units in Manhattan is subject to rent-stabilization. The fraction was 58 percent in 2011. At the same time, the average rent was \$2,640 in the unregulated market and \$1,317 in the regulated market.

Households tend to stay for long periods in their apartments. On average in 2011, households had occupied their apartments 16.18 years for public housing and 9.49 years for rent-stabilized housing. The turnover is much higher in the unregulated housing market. Not surprising, households in public housing are much poorer than household in rent stabilized and unregulated housing. Families that live in public housing tend to be single-parent households, the majority headed by a female. Public housing families have more children, on average, than households in rent-stabilized or unregulated housing.

Finally, we rely on the 2008 Survey of Income and Program Participation (SIPP) to estimate the autocorrelation of the income process. Note that the NYHVS is a repeated cross-sectional data set and not a panel. Our SIPP sample consists of households that live in a metropolitan area in the State of NY.

3 A Dynamic Model of Affordable Housing Markets

3.1 The Baseline Model

We consider a local housing market with three housing options: public housing (p), rent-regulated housing (r), and housing provided by the unregulated market (m). The exogenous housing stocks in public and rent-regulated housing are given by k_p and k_r .¹⁵

¹⁵The assumption of a fixed supply of public and rent-stabilized housing is appropriate for NYC during our sample period. There was limited recent construction of new housing communities or rent-stabilized housing units in NYC. If anything, the supply of rent-stabilized housing has declined in the past decades. We discuss these issues in more detail in Section 6 of the paper.

Time is discrete, $t = 0, \dots, \infty$. Households are infinitely lived and forward-looking. Households have a common discount factor β and maximize expected lifetime utility. In the baseline model, households only differ by income, denoted by y . Income evolves according to a stochastic law of motion that can be described by a stationary Markov process with transition density $f(y'|y)$. Below we extend our model to allow for additional sources of household heterogeneity.

Changes in housing consumption are subject to adjustment costs. Households that either move across three housing sectors or change the level of housing within the unregulated market incur a utility cost of γ . The previous choice of the housing sector $d \in \{p, r, m\}$ and the level of housing service in the unregulated market \bar{h} are, therefore, relevant state variables in this model.

The flow utility is defined over housing quality, h , and a numeraire good, b . Consider a household that rents in the unregulated market. Housing services can be purchased at price p_m .¹⁶ Flow utility is, therefore, given by:

$$u_m(y) = \max_{h,b} U(b, h) \tag{1}$$

$$s.t. \quad p_m h + b = y$$

Note that we are imposing the realistic assumption that low- and moderate-income households do not save and cannot borrow against uncertain future income. They are liquidity constrained and spend their incomes on housing and consumption goods in each period.

Households in the unregulated market can choose to stay in the same house and continue to consume the housing service \bar{h} as in the previous period. The flow utility is given by:

$$u_s(y, \bar{h}) = U(y - p_m \bar{h}, \bar{h}) \tag{2}$$

There are R discrete different levels of housing quality in the stabilized market. The flow

¹⁶We implicitly assume that unregulated housing supply is perfectly elastic at price p_m . This assumption can be easily relaxed to endogenize the price of housing in the unregulated market by allowing for an upward sloping supply function.

utility associated with a rent-regulated unit of quality h_r and price $p_r < p_m$ is given by:

$$u_r(y) = U(y - p_r h_r, h_r) \quad r = 1, \dots, R \quad (3)$$

The next assumption captures the search frictions in that market.

Assumption 1

- a) *Each period, there is a positive probability q_r that a household receives an offer to move into a rent-regulated unit of quality h_r .*
- b) *Each household receives, at most one, offer per period.*

The probabilities of receiving an offer to move into a stabilized housing unit are endogenous and depend on the supply and the voluntary outflow from regulated housing as discussed below in detail.

To simplify the notation we set $R = 1$ for the remainder of this section. All results can be easily generalized to account for heterogeneity in the quality and supply of rent-stabilized units.¹⁷ In our quantitative analysis below, we estimate a model with two discrete types of stabilized units.

Public housing provides a constant level of housing consumption, h_p . Households pay rents that are proportional to income. Hence, rents are equal to τy , where τ is the "income tax rate" associated with public housing. In practice, τ is approximately 0.3 for most housing communities and cities. Per period utility in public housing is, therefore, given by:

$$u_p(y) = U((1 - \tau) y, h_p) \quad (4)$$

The local housing authority that administers the public housing program manages a wait list. The priority score of a household is a monotonic function of the time spent on the wait list. More formally, let w denote the time that a household has been on the wait list. Let $p(w)$

¹⁷An appendix that contains a detailed derivation of all key equations is available upon request from the authors.

denote the probability that a household who has been on the waitlist for w periods receives an offer to move into public housing. The next assumption captures the behavior of the housing authority.

Assumption 2

- a) The housing authority makes take it or leave it offers. If a household rejects an offer, the priority score is reset to zero ($w = 0$).*
- b) The outflow of public housing is voluntary, and the housing authority does not evict households from public housing.*
- c) Eligibility is determined by an income cut-off, denoted by \bar{y} and is checked every time period. Loss of eligibility means that the priority score is reset to zero ($w = 0$).*

These assumptions reflect a common practice of housing authorities in NYC and other U.S. metropolitan areas. Note that the distribution of priority scores is endogenous and determined in equilibrium as we discuss below.

The timing of decisions is as follows:

1. Each household gets a realization of income which determines the income distributions at the beginning of the period.
2. Some households get an offer to move into public housing. These offers are generated with probability $p(w)$.
3. Some households get an offer to move into rent-regulated housing unit of quality r . These offers are generated with probability q_r .
4. Households decide whether to move or not. Households in the private sector determine their housing consumption level. All households obtain the flow utility that depends on their decisions.
5. Priority scores are updated.

Note that the flow utility is realized after households have relocated.

The four state variables in this model are the priority score, w , the income, y , the previous housing sector, d , and the previous level of housing service, \bar{h} . Define the conditional value functions associated with the four choices:

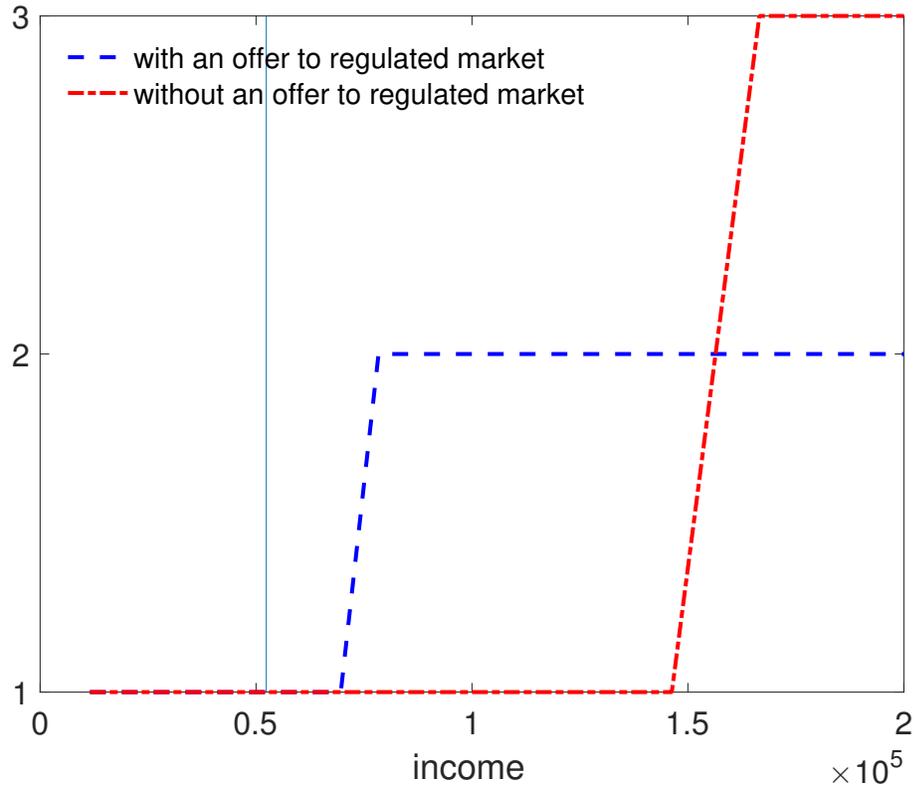
$$\begin{aligned}
v_p(y, d) &= u_p(y) + \gamma 1\{p \neq d\} + \beta \int V_p(y') f(y'|y) dy' \\
v_r(y, d, w) &= u_r(y) + \gamma 1\{r \neq d\} + \beta \int V_r(y', w) f(y'|y) dy' \\
v_m(y, w) &= u_m(y) + \gamma + \beta \int V_m(y', \bar{h}', w) f(y'|y) dy' \\
v_s(y, \bar{h}, w) &= u_s(y, \bar{h}) + \beta \int V_m(y', \bar{h}, w) f(y'|y) dy'
\end{aligned} \tag{5}$$

Note that we have imposed the condition that $V_s(y, \bar{h}, w) = V_m(y, \bar{h}, w)$. Also note that \bar{h}' is a deterministic function of the current income, y . We can derive recursive expressions for the unconditional value functions. The value functions, which are characterized in detail in Appendix A, then determine the optimal decision rules for each household.

To illustrate the optimal decision rules we consider an estimated specification of the model with only one type of rent-stabilized housing. Figure 1 plots the policy function for a household in public housing. The blue vertical line indicates the income eligibility threshold for public housing. Optimal decision rules can be characterized by threshold rules. The blue line indicates the decision rule of a household that received an offer to move into regulated housing while the red line is a household without an offer.

Low- and moderate-income households prefer to live in public housing, moderate-income households prefer rent-regulated housing while high-income households prefer renting in the unregulated market. Note that households in public housing are more likely to voluntarily move out of public housing if they have an offer to move into a rent-regulated apartment. Of course, the thresholds depend on the parameter values of the model. It is possible to generate specifications of the model in which high-income households want to move into private housing. Similarly, we can characterize the decision rules for households in private or rent-stabilized

Figure 1: Policy Function (1=public, 2=regulated, 3=unregulated)



housing.

To finish characterizing the equilibrium, we need to define the inflows and outflows that are generated by our model. Let $g_m(w)$ ($g_r(w)$) denote the marginal distribution of wait times for households in unregulated (rent-regulated) housing in stationary equilibrium. Let $g_p(y)$ denote the density of income of households that are inside public housing at the beginning of each period (before households have moved). Let $g_m(y, \bar{h}|w)$ denote the stationary density of current income and previous housing service conditional on wait time for households in the unregulated market. Similarly, let $g_r(y|w)$ denote the stationary density of income conditional on wait time for households in the regulated market.¹⁸

¹⁸We characterize the laws of motions for these densities in detail in the appendix of this paper.

Given these densities, we can characterize the resulting inflows and outflows for public and rent-stabilized housing. Consider the voluntary flow of households out of public housing. It is given by:

$$\begin{aligned}
OF_p &= k_p (1 - q_r) \int 1\{v_m(y, 0) > v_p(y, p)\} g_p(y) dy \\
&+ k_p q_r \int 1\{v_m(y, 0) \geq \max[v_p(y, p), v_r(y, p, 0)]\} g_p(y) dy \\
&+ k_p q_r \int 1\{v_r(y, p, 0) \geq \max[v_p(y, p), v_m(y, 0)]\} g_p(y) dy
\end{aligned} \tag{6}$$

Note that the first two terms are outflows to the unregulated market and the third term captures the outflow to the rent-regulated market. The flow into public housing is given by:

$$\begin{aligned}
IF_p &= k_m \sum_{j=0}^{\infty} p(w_j) g_m(w_j) IF_{mp}(w_j) \\
&+ k_r \sum_{j=0}^{\infty} p(w_j) g_r(w_j) IF_{rp}(w_j)
\end{aligned} \tag{7}$$

The inflow from the unregulated market conditional on wait time is:

$$\begin{aligned}
IF_{mp}(w_j) &= (1 - q_r) \int \int_{y \leq \bar{y}} 1\{v_p(y, m) \geq \max[v_m(y, 0), v_s(y, \bar{h}, 0)]\} g_m(y, \bar{h}|w_j) dy d\bar{h} \\
&+ q_r \int \int_{y \leq \bar{y}} 1\{v_p(y, m) \geq \max[v_r(y, 0), v_m(y, 0), v_s(y, \bar{h}, 0)]\} g_m(y, \bar{h}|w_j) dy d\bar{h}
\end{aligned} \tag{8}$$

Similarly, we can characterize the flows for rent-regulated housing, denoted by IF_r and OF_r , as discussed in detail in Appendix C of this paper.

In a stationary equilibrium, the inflows have to be equal to the outflows for public and rent-regulated housing.¹⁹

Definition 1 *A stationary equilibrium for this model consists of the following: a) offer probabilities $p(w)$ and q_r , b) distributions $g_p(y)$, $g_m(w)$, $g_r(w)$, $g_m(y, \bar{h}|w)$, and $g_r(y|w)$, and c) value functions $V_p(y)$, $V_m(y, \bar{h}, w)$ and $V_r(y, w)$, such that:*

¹⁹The vacancy rate in NYC has been around 2 percent during the time period of interest. Hence we ignore vacancies.

1. *Households behave optimally and value functions satisfy the equations above.*
2. *The housing authority behaves according to the administrative rules described above.*
3. *The densities are consistent with the laws of motion and optimal household behavior.*
4. *$p(w)$ satisfies the market clearing condition for public housing:*

$$OF_p = IF_p \tag{9}$$

5. *q_r satisfies the market clearing condition for rent-regulated housing:*

$$OF_r = IF_r \tag{10}$$

Finally note that we can endogenize the price of housing in the unregulated market by assuming that there is an upward sloping housing supply function $H_m^s(p_m)$. The demand for unregulated housing given by:

$$H_m^d = (1 - k_p - k_r) \sum_j g(w_j) \int h(p_m, y, \bar{h}) g_m(y, \bar{h} | w_j) dy d\bar{h} \tag{11}$$

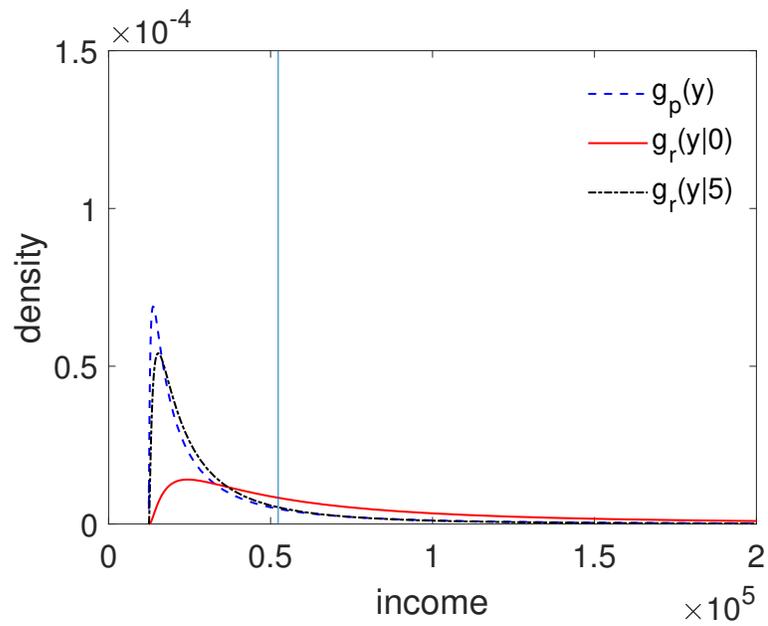
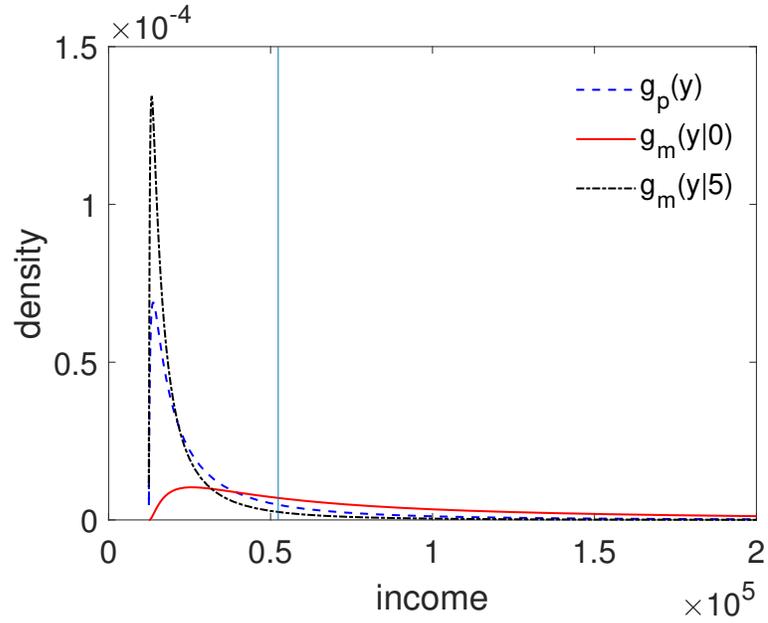
Imposing the market clearing condition for unregulated housing then endogenizes p_m . We use this approach in some of our counterfactual policy experiments.

Figure 2 illustrates the stationary equilibrium densities of income for a specification of our estimated model with one household type and only one type of rent-stabilized housing.

The top panel compares the marginal income densities of households in the unregulated market with densities of households that are in public housing.²⁰ We find that households with a priority score of zero, who are ineligible for public housing, have a much higher income than those who live in public housing. Note that households with a priority score of five years have lower incomes, on average. This due to the fact that households with high priority scores must have had incomes below the eligibility threshold for a number of consecutive periods

²⁰The marginal density of income of households in the private market is given by $\int g_m(y, \bar{h} | w) d\bar{h}$.

Figure 2: Stationary Distributions



to remain eligible, while this criterion does not apply for households in public housing. The lower panel compares the income density of households in the rent-stabilized market with the income density of households in public housing. Again we find similar qualitative patterns. Households that live in stabilized housing with high priority scores look similar to households in public housing.

Next we characterize the properties of equilibria with rationing. The main analytical result is summarized by the following proposition.

Proposition 1 *Any stationary equilibrium with sufficiently strong excess demand for public housing has the property that there exists a value $\bar{w} < \infty$ such that: $p(\bar{w}+1) = 1$, $0 \leq p(\bar{w}) < 1$, and $p(\bar{w} - j) = 0$ for all $j \geq 1$.*

Note that $p(\bar{w} + 1) = 1$ implies that there are no households with priority score greater than $\bar{w} + 1$, i.e. $g(\bar{w} + 1 + j) = 0$, for $j \geq 1$.

Proof:

We use a proof by contradiction. Suppose not, then

$$p(\bar{w} + 1) < 1 \tag{12}$$

and next period there exists some households with priority score $\bar{w} + 2$, hence $g(\bar{w} + 2) > 0$ which violates the stationarity definition and the definition of \bar{w} .

Suppose that $p(\bar{w} - j) > 0$ and $p(\bar{w}) \leq 1$. This case violates the assumption that offers to households with lower priority ranks can only be made if all households with higher ranks receive offers.

Suppose that $p(\bar{w} - j) > 0$, $p(\bar{w}) = 1$ and $p(\bar{w} + 1) = 1$, then there will be no household in the next period which priority score $\bar{w} + 1$ which violates the stationarity assumption and that and that $g(\bar{w} + 1) > 0$.

Q.E.D.

The equilibrium has the property that everybody in the highest priority group obtains an

offer to move into public housing. In addition, a fraction of the households with the second highest priority also gets an offer. The remaining households with the second highest priority score who do not get an offer this period will obtain an offer in the next period. The intuition for this result is the following. The wait list partitions the potential demand into $\bar{w} + 2$ cohorts. By adjusting $p(\bar{w})$, we can smooth out the fraction of individuals that obtains an offer. Note that $p(w_j)$ is not uniquely defined for $w_j > \bar{w} + 1$. Since the housing authority makes take-it-or-leave-it offers, there will be no households with wait times larger than $\bar{w} + 1$. Without loss of generality, we can, therefore, set $p(\bar{w} + j) = 1$ for all $j > 1$.

Given this equilibrium offer function, the inflow into public housing has two components and is equal to:

$$\begin{aligned}
 IF_p &= p(\bar{w}) [k_m g_m(\bar{w}) IF_{mp}(\bar{w}) + k_r g_r(\bar{w}) IF_{rp}(\bar{w})] \\
 &+ [k_m g_m(\bar{w} + 1) IF_{mp}(\bar{w} + 1) + k_r g_r(\bar{w} + 1) IF_{rp}(\bar{w} + 1)]
 \end{aligned} \tag{13}$$

To finish the characterization of the equilibrium, we need to provide the laws of motion for the equilibrium densities. Equations (20) - (28) in Appendix B provide the details.

3.2 Extending the Model to Allow for Multiple Household Types

Households differ across many observed attributes besides income, such as family size, race, ethnicity or gender of the household head. We extend our model to capture these differences using discrete household types, which also allows us to include differences in preferences over public housing.²¹

We assume that there are I mutually exclusive types of households. Household types are defined by observed discrete characteristics (number of kids, number of adults etc.) Each

²¹This approach could also be used to capture the fact that some households may not consider public housing a desirable housing choice due to its stigma as suggested by Moffitt (1983).

household has a fixed share denoted by s_i , where $\sum_{i=1}^I s_i = 1$.²² We make the following simplifying assumption which can be easily relaxed.

Assumption 3 *The housing authority operates one wait list for all types and all types compete for the same housing units in the unregulated and regulated markets.*

Given that we do not have any data characterizing different wait lists, this a natural starting point for the analysis. We should point out that this assumption can be easily relaxed. We have also estimated versions of the model with heterogeneity that account for separate wait lists for households with different characteristics and obtained similar results. As we show in Appendix D, we derive the value functions, in- and outflows, as well as the laws of motions for the key densities for this extended model. The derivations are similar to the ones discussed for the baseline model without different household types.

4 Identification and Estimation

Since equilibria can only be computed numerically, we introduce a parametrization of the model in this section. We then discuss identification and estimation.

There are four choices in our model: public housing p , rent-stabilized housing r , move to a new unregulated unit m , stay in the same unregulated unit s . We assume that the flow utility functions can be approximated by a Stone-Geary utility function. Hence, the four flow utilities

²²This approach is in the spirit of Heckman and Singer (1984) although we will treat the household type as observed.

are given by:

$$\begin{aligned}
u_p(y, h_p) &= (h_p - \underline{h})^\alpha [(1 - \tau)y]^{(1-\alpha)} \\
u_r(y, h_r) &= (h_r - \underline{h})^\alpha [y - p_r h_r]^{(1-\alpha)} \\
u_m(y) &= \alpha^\alpha (1 - \alpha)^{1-\alpha} (y - p_m \underline{h}) p_m^{-\alpha} \\
u_s(y, \bar{h}) &= (\bar{h} - \underline{h})^\alpha [y - p_m \bar{h}]^{(1-\alpha)}
\end{aligned} \tag{14}$$

Recall that α determines the housing share parameter if $\underline{h} = 0$. The parameter \underline{h} captures minimum housing consumption. Adding this parameter, therefore, implies that our model predicts that housing shares are decreasing in income. Both parameters are, therefore, identified based on the observed housing shares by income.

We fix the annual discount factor, β , at 0.95. We normalize the price of housing in the unregulated market, p_m , to be equal to one since the units of housing services are arbitrary. The tax rate in public housing, τ , is determined by the administration of public housing programs. It is a state policy that renters in public housing pay 30 percent of their income in rent.²³

To identify and estimate the price discount in the rent-regulated market, p_r , we assume that market rents can be decomposed into a price and a quality index. We assume that the quality index is the same for units in the unregulated and the regulated markets, but the prices are not. We can, therefore use the techniques discussed in Sieg, Smith, Banzhaf, and Walsh (2002) to identify and estimate the price discount in the regulated market.

The quality parameters h_r for $r = 1, \dots, R$ are identified based on our classification algorithm discussed below and the observed market rents for each type of unit type conditional on observed characteristics.

Public housing quality, h_p , is identified from the observed demand for public housing, such as average income of households in the public housing and average time spent in public housing. As can be seen in Figure 1, a household whose income is below certain cutoff chooses public

²³Also note that k_m , k_p and k_r are observed in the data.

housing if offered. The cutoff is an increasing function of the public housing quality. The model predicts that the average income of households in public housing is increasing functions of the public housing quality. Similarly, with the higher cutoff, households are less likely to move out of the public housing, thus average time spent in the unit is also an increasing function of the public housing quality.

Income consists of a fixed minimum income $\underline{y} = p_m \underline{h}$ plus a stochastic component, \tilde{y}_{it} . Hence, we have $y_{it} = \underline{y} + \tilde{y}_{it}$. We assume that the logarithm of stochastic income for each household follows an AR(1) process:

$$\ln(\tilde{y}_{it}) = \mu + \rho \ln(\tilde{y}_{it-1}) + \epsilon_{it}^y \quad (15)$$

where the standard deviation of the error term is given by σ . The mean and the standard deviation of log income are identified of the observed income distributions in the data. We use panel data from the Survey of Income and Program Participation to estimate the autocorrelation parameter of the income process.

All parameters of the income process and household preferences depend on household type in the extended model. The household type is observed by the econometrician. The identification argument, therefore, extends to that model since all relevant moments are observed conditional on type.

The introduction of moving costs generates a more realistic model. Previous empirical housing studies have suggested that moving costs may be substantial. Adding these frictions generates persistence in housing choices in the non-stabilized market. As a consequence, the moving cost parameter γ is primarily identified from the observed length of stay in an apartment that is not subject to rent-stabilization.

The arguments for identification are constructive and suggest that we can estimate the parameters of our model using a method of moments estimator. We use the following moments in estimation: the fraction of each housing type, the average time spent in the unit by housing type, the average income by housing type, the variance of income by housing type, the average

rents by housing type. Asymptotic standard errors can be consistently estimated using the standard formula for a parametric method of moments estimator provided, for example, in Newey and McFadden (1994).

5 Empirical Results

5.1 Measuring the Discount in Rent-Stabilized Housing

To measure the relative price between unregulated and regulated housing, we estimate a hedonic regression using data on housing units in both market. As discussed in Section 4, we assume that the quantity index that relates structural and neighborhood characteristics to housing service flows is constant among the two markets. We can, therefore, use these regressions to measure price differences between regulated and unregulated housing markets.

The regression also includes dummy variables that indicate whether the building has an elevator, the building age, the building size, a dummy for the fuel type, a dummy for condo/coop, a dummy for bad walls and a unit floor control. Table 2 summarizes our findings based on regressions with and without sub-borough controls.

Column I reports the results without sub-borough fixed effects. This regression suggests that regulated units are, on average, 62 percent cheaper in Manhattan compared to the market rated units. Once we allow for sub-borough fixed effect, the estimated discount drops to 51 percent. We use the more conservative estimate of the discount for rent-stabilized housing in the analysis below.

Estimating this price differential requires the assumption that rent-regulated and unregulated housing do not differ based on unobservables. There are some obvious concerns regarding this assumption. Rent-regulated units can be poorly maintained. Fortunately, NYCHVS provides very detailed information about the physical characteristic of the housing units. In our

Table 2: Log Rent Regression

	I	II
regulated	-0.621*** (0.027)	-0.513*** (0.026)
# of bed rooms	0.066*** (0.014)	0.124*** (0.014)
# of other rooms	-0.013 (0.021)	-0.002 (0.020)
complete kitchen	0.361** (0.151)	0.370*** (0.139)
complete plumbing	0.790*** (0.213)	0.622*** (0.197)
constant	7.031*** (0.596)	7.188*** (0.549)
passenger elevator in building	Yes	Yes
building age	Yes	Yes
building size	Yes	Yes
building condition	Yes	Yes
fuel type	Yes	Yes
condo/coop	Yes	Yes
bad walls	Yes	Yes
floor of unit	Yes	Yes
sub-borough control	No	Yes
# of observations	1,416	1,416
R-squared	0.465	0.553

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

hedonic regression, we control for the condition of the building (dilapidated, deteriorated, sound) condition of the exterior wall (existence of major crack, etc), and age of the building along with typical housing quality controls. Furthermore, we also include a sub-borough identifier (dividing the Manhattan borough to 10 sub-regions) to control for potential heterogeneity in neighborhood quality.

Using the estimated hedonic model we can predict the quality of each apartment in our sample. 25th and 75th percentiles of the distribution are \$24,963 and \$34,239 respectively. To reduce the computational complexity of the model, we approximate this continuous distribution using two discrete types. We define housing quality as low (high) if the predicted quality index of the housing unit is below (above) the median quality.²⁴

5.2 Parameter Estimates and Goodness of Fit

Table 3 reports the estimated parameters and standard errors for three specifications of our model. First, we estimate the baseline model. Second, we add heterogeneity in regulated housing types. Finally, we add heterogeneity in discrete household types. Overall, we find that all parameters are estimated with good precision.

Consider the baseline model in Column I. The parameter α captures the housing expenditure share for households with sufficiently high income such that the impact of the minimum housing consumption \underline{h} is negligible. In our application, these are household with incomes of approximately \$200,000 and more. Low-income households tend to have higher housing expenditure shares than high-income households. This is captured by our estimate of \underline{h} which is approximately \$12,395. Once we allow for heterogeneity in regulated housing types in Column II, our estimate of α increases to 0.28, and the estimate \underline{h} decreases to \$10,480. As a consequence, low- and moderate-income households spend approximately 40 to 45 percent of income on housing if they rent in the unregulated market. High-income households spend

²⁴Appendix E shows the histogram of the housing quality in the regulated market.

Table 3: Estimated Parameters

	I	II	III	
	Baseline	1 Household Type	2 Household Type	
	all	all	female	male
α	0.23 (0.007)	0.28 (0.009)	0.27 (0.017)	0.30 (0.020)
h	12,395 (1270)	10,480 (149)	10,949 (54)	10,439 (53)
γ	14,014 (40)	11,081 (390)	9,827 (1795)	12,223 (88)
μ_y	9.62 (0.038)	9.74 (0.020)	9.80 (0.025)	9.79 (0.037)
σ	1.12 (0.007)	1.07 (0.008)	1.06 (0.011)	1.01 (0.011)
ρ	0.67 (0.02)	0.67 (0.02)	0.63 (0.03)	0.70 (0.03)
h_p	17,912 (169)	18,069 (203)	19,177 (257)	
h_r	32,172 (219)			
h_r^1		27,499 (378)	28,436 (231)	
h_r^2		34,467 (290)	35,346 (368)	

Standard errors are in parenthesis.

approximately 30-35 percent of income on housing. We also find that there are substantial moving costs, which are captured by the parameter γ . The estimate is \$14,014 (\$11,081) in Column I (II).

Column III shows that female-headed households have slightly lower estimates of α and higher estimates of \underline{h} than male-headed households. Female-headed households also tend to have lower moving costs than male-headed households. None of these differences are, however, significant at conventional levels. The parameters of the income process, however, depend on the observed household type. Female-headed households have more volatile and less persistent incomes than male-headed households.

Housing quality is measured as equivalent expenditures in the unregulated market. The baseline estimate in Column I shows that an average public housing unit in Manhattan provides the same quality as a unit that rents for \$17,912 dollars in the unregulated market. The average quality of rent-stabilized housing in the baseline model is \$32,172. Allowing for heterogeneity in rent-stabilized housing in Column II indicates that the quality for a low (high) quality rent-stabilized apartment is \$27,499 (\$34,467). Allowing for heterogeneity in household types does not affect these estimates. We thus conclude that public housing units are of significantly lower quality than low-quality, rent-stabilized housing units. High-quality, rent-stabilized units provide, on average, 25 percent more housing services than low-quality units.

Table 4: Properties of Equilibrium

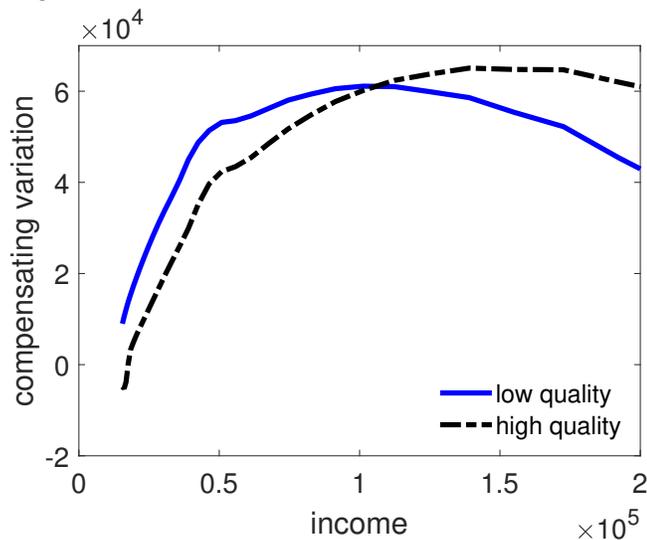
		Baseline	1 Household Type	2 Household Type
wait	\bar{w}	20	19	19
times	$p(\bar{w})$	0.77	0.19	0.65
search	q_1	0.26	0.11	0.10
frictions	q_2		0.26	0.22

Table 4 summarizes some properties that correspond to the equilibria that are implied by

the parameter estimates. Our model generates wait times of approximately 19 to 20 years. The probability of finding a rent-stabilized unit is approximately 25 percent for high-quality units and 10 percent for low-quality units. Overall, these predictions are quite plausible.

Our estimates imply that affordable housing is an attractive option for low- and moderate-income households in Manhattan. To illustrate the magnitude of these effects, we compute compensating variations that measure the benefits of having access to rent-stabilized housing relative to renting in the unregulated market. Figure 3 plots the compensating variations by income for the two quality levels of rent-stabilized housing, which validates our estimation strategy and model specification.

Figure 3: The Benefits of Rent-Stabilized Housing



Our analysis suggests that even low-quality units of affordable housing are quite attractive. A household with approximately \$100,000 gains approximately \$60,000 from having access to a low-quality, rent-stabilized units. High-quality units are also attractive. A household with approximately \$150,000 gains approximately \$65,000 from having access to a high-quality, rent-stabilized unit. Figure 3 shows that there is an inverted-u shaped relationship between income and welfare gains. As a consequence, rent-stabilization policies create a fair amount of

misallocations in housing markets.²⁵

Given these large benefits associated with affordable housing, it is not surprisingly that rent-stabilization and affordable housing policies are popular at the ballot box, not only with low- and moderate-income households, but also with higher-income households that rent. As a consequence, our model explains the prevalence and political popularity of affordable housing policies in places such as NYC and Los Angeles.

We focused in this section on the Manhattan subsample. We also estimated specifications of the model using the full sample that includes renters of all five boroughs. The results are qualitatively similar to the results reported above. The main difference is that the price discount for affordable housing, wait and search times are lower using data from all five boroughs.

Tables 5 reports a variety of goodness of fit statistics. We report the key statistics for the models in Columns I and II. Overall, we find that our model fits the key moments used in estimation well.²⁶

6 Policy Analysis

6.1 Increasing the Supply of Rent-stabilized Housing

The popularity of affordable policies is in stark contrast to longer term trends in the supply of affordable housing in NYC. Landlords primarily bear the burden of the rent-stabilization policies. Note that less than 8 percent of all apartments in NYC are voluntarily rent-stabilized. These landlords obtain significant tax breaks as a return for making a fraction of the housing units affordable. The vast majority of rental units in NYC – approximately 92 percent – are “involuntarily” stabilized under New York State’s Rent-stabilization Law. In NYC landlords

²⁵Glaeser and Luttmer (2003) find that 21 percent of New York apartment renters live in units with more or fewer rooms than they would if they rented in the unregulated market in 1990.

²⁶The goodness of fit statistics for the model on Column III are similar to the one shown in Table 5.

Table 5: Model Fit

housing	years		income		market rent		income rage
Baseline							
	data	model	data	model	data	model	
Public	16.18	17.57	32930	30405	—	—	
Regulated	9.49	9.41	54739	57983	1317	1307	
Market	3.85	2.64	71045	70045	—	—	
					2581	2444	$75000 \leq y < 100000$
					2895	2951	$100000 \leq y < 125000$
					3430	3477	$125000 \leq y < 150000$
1 Household Type							
Public	16.18	16.81	32930	28872	—	—	
Regulated1	9.18	10.04	49651	49326	1071	1117	
Regulated2	9.81	6.13	59859	63588	1570	1400	
Market	3.85	2.17	71045	69673	—	—	
					2581	2627	$75000 \leq y < 100000$
					2895	3088	$100000 \leq y < 125000$
					3430	3655	$125000 \leq y < 150000$

have long been allowed to deregulate vacant apartments if the legal rent for a new renter exceeds a threshold, which is currently \$2,700 a month. Between 1993 and 2015 more than 139,000 apartments have been converted to market rates through vacancy decontrol which has led to a significant decline in the supply of affordable housing (WSJ, 2015).²⁷

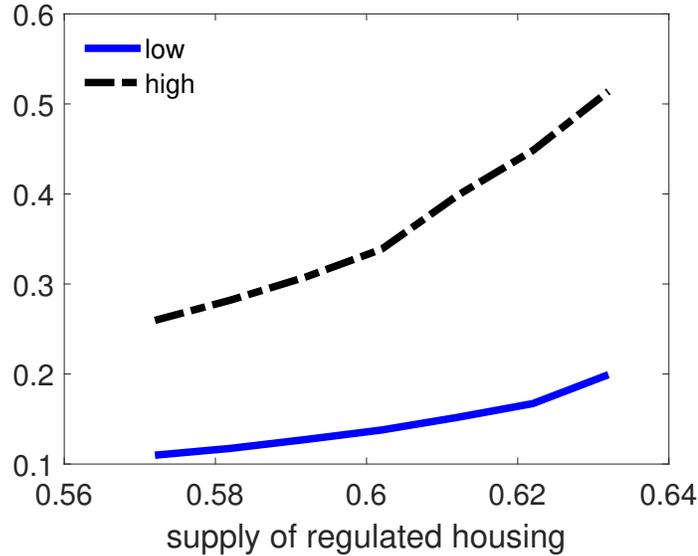
Not surprisingly, these trends have not gone unnoticed. As a candidate, the current mayor of NYC, Bill de Blasio, successfully ran on a platform that promised significant increases in the provision of affordable housing. Once in office, he proposed and city council recently adopted a 10-year plan to build and retain 200,000 affordable housing units in the NYC area through various rezoning laws. We can use our model to simulate the effects of these types of policy changes on renters' welfare. Using our model with two affordable housing types, we increase the supply of affordable housing by up to 10 percent. We consider the impact of this policy change on the probability of finding an affordable housing unit, the wait time for public housing, as well as the distribution of renters' welfare in the economy.

Here we consider the impact of increasing the supply of regulated housing from 57.2% to 63.2% of total rental units in Manhattan. We increase the supply of low-quality and high quality regulated housing equally. Figure 4 shows that a ten percent increase in the supply of affordable housing substantially increases the probability of receiving an offer to move into a regulated housing unit in equilibrium. The probability of finding a low-quality unit increases from 10 to 20 percent, while the probability of finding a high-quality unit increases from 25 to 50 percent. Wait times for public housing also decrease by up to 1 year.

The reduced wait and search times are associated with a more efficient allocation of public and rent-stabilized housing in equilibrium. Households are more likely to move out of affordable units when they receive positive income shocks. Hence, those units can be reallocated faster to more needy households. As a consequence, the time spent in public or regulated housing decreases significantly. Similarly, households in the unregulated market also spend less time in

²⁷The NYCHVS suggests that more than 70 percent of all renters in Manhattan with incomes less than \$200,000 live in a rent-stabilized unit in 2002.

Figure 4: Offer Probability (p_r)



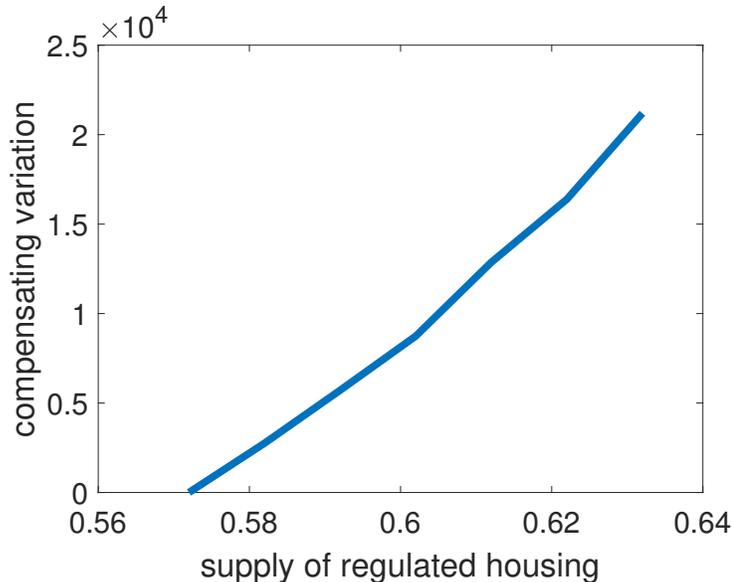
the unit because of the reduced wait and search times for affordable housing.²⁸

In a static framework subsidized housing is primarily occupied by low-income households. An increase in the supply of subsidized housing implies that the average income of households in those subsidized units increases as more moderate-income households move into those units. In our dynamic framework, there is another countervailing effect. As the allocation of affordable housing becomes more efficient, those units are occupied by households whose realized income is low due to negative income shocks. As we increase the supply of regulated housing we find that the average income of households in public housing and low-quality regulated housing decreases.

We can measure the welfare gains for households using compensating variations. Figure 5 plots the average welfare gain as a function of the increase in the regulated housing stock. We find that the average welfare gain of a 10% increase in affordable housing is approximately

²⁸Another measure of the inefficiency of the equilibrium allocation is the fraction of ineligible household in public housing. We find that increasing the supply of regulated housing can also mitigate this problem to some degree.

Figure 5: Change in Welfare



\$20,000. Note that all households in our model benefit from a permanent increase in rent-stabilized housing. The lower wait and search times imply that all households are better insulated against negative future income shocks. Overall, the welfare gains from this insurance are larger for high-income households. Of course, the biggest gains are for those who prefer rent-stabilized housing and now are more likely to obtain access to it.

Finally, we evaluate how sensitive the results of our policy analysis are to changes in the supply of unregulated housing. In the baseline model, we assume that the housing supply for unregulated housing is perfectly elastic. Hence, prices in unregulated housing are not affected by the change in the supply of affordable housing. Alternatively, we can use an aggregate housing supply function given by $H_m^s(p_m) = l [p_m]^\epsilon$. We set the constant l such that demand and supply are equal when $p_m = 1$ in our baseline year 2011. To evaluate the robustness of our findings we repeated the exercise above assuming a variety of different values for the supply elasticity of unregulated housing. Here we focus on the case when $\epsilon = 0.5$. A ten percent increase in regulated housing reduces the demand for unregulated housing. As a consequence, the rental price for unregulated housing drops by 10 percent. As private housing gets cheaper,

it becomes more attractive. As a consequence, the reduction in waiting and search times are even steeper than in the baseline model. But overall, we obtain the same qualitative and quantitative results.²⁹

6.2 Taxing Ineligible Households in Public Housing

One notable feature of existing public housing policies is that housing authorities rarely ask a household to leave public housing once its income exceeds the eligibility threshold. Approximately 17 percent of households living in public housing in Manhattan have incomes that exceed 80 percent of the median income and are, thus, likely to be ineligible of housing aid. For these households, housing aid is de facto an open-ended entitlement program.

Since evictions are politically not feasible, we consider a different mechanism to encourage higher income households to leave public housing. Currently, the housing authority uses a flat income tax to generate rental income. The tax rate is 30 percent. The basic idea is to replace the current flat tax with a revenue-neutral progressive tax system. Here we explore a piecewise linear tariff with a low marginal rate, denoted by τ , for incomes below the eligibility threshold (\bar{y}) and a higher rate, denoted by $\tau + \Delta\tau$, for incomes above the threshold. Hence, the new tax function is given by

$$T_h(y) = \tau y + \Delta\tau \max[y - \bar{y}, 0] \quad (16)$$

For a given value of τ we can solve for the revenue-neutral value of $\Delta\tau$, using a simple line-search algorithm.

Figure 6 illustrates revenue-neutral combinations of $\Delta\tau$ and τ in equilibrium. Figure 7 captures the welfare implications of these policy changes. Overall, we find that the progressive revenue-neutral tax is welfare improving. Welfare gains are up to \$1400 per household when we set $\tau = 0.2$ and $\Delta\tau = 0.28$. The more progressive the tax system, the higher the aggregate

²⁹Details are available upon request from the authors.

Figure 6: Revenue-neutral Combinations of $\Delta\tau$ and τ

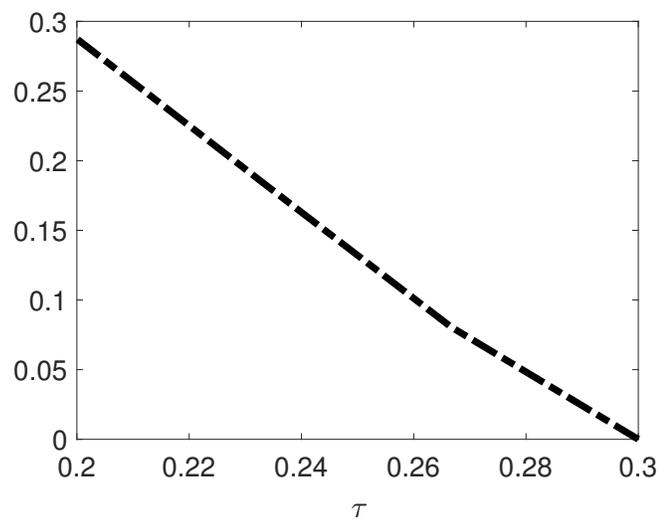
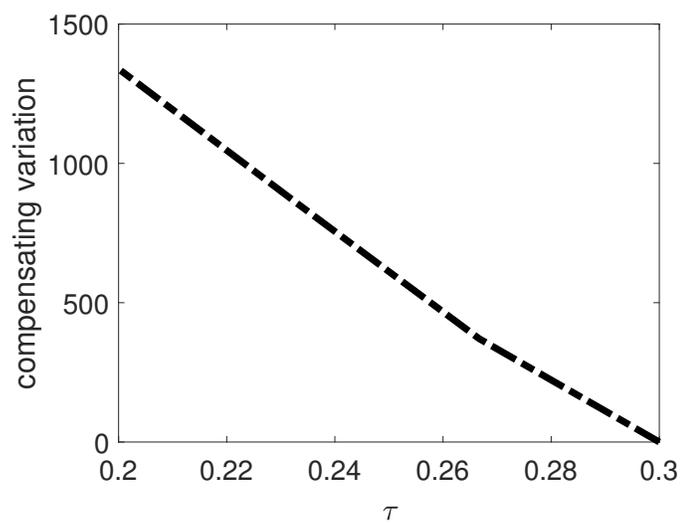


Figure 7: Changes in Aggregate Welfare



welfare gains.

However, these revenue-neutral policies do not reduce wait times. The main reason for this result is that the progressive tax schedule provides even stronger insurance against negative income shocks. Thus, anticipating future negative income shocks, households are even less willing to move out of public housing.

To reduce wait times we need to consider policies that are not revenue-neutral. For example, we can fix τ at the current level of 0.3 and then increase $\Delta\tau$. This policy makes public housing less attractive for low- and moderate-income households. It reduces the average waiting times and generates additional revenues. However, it does not necessarily improve welfare. For example, if we set $\Delta\tau = 0.2$, we can reduce the waiting time of the public housing by 1.1 years. However, the average welfare of the renters will drop by \$300 on average.

7 Conclusions

We have developed a new dynamic model that captures search and queuing frictions in the rental markets for affordable housing, as well as mobility cost. We have characterized the stationary equilibrium with rationing that arises in the model.³⁰ We have shown how to identify and estimate the structural parameters of the model. Our application focuses on the housing markets of Manhattan in 2011. Overall, our model fits the observed sorting of households well. We have characterized the distribution of welfare that arises in our model. We have shown that access to low (high) quality of affordable housing can increase welfare by as much as \$60,000 (\$65,000). As a consequence, our model provides a compelling explanation of why affordable housing policies have been popular with the vast majority of urban renters in NYC. Finally, we have studied the effects of expanding the supply of affordable housing and

³⁰Stationary is always a simplifying, but often necessary assumption. There is no evidence of “bubbles” in rental markets during our time period. Rental markets tend to be much less volatile than markets for owner-occupied housing.

the effects of progressively taxing households in public housing. We find that both policies can improve overall welfare.

We should point out that we cannot conclude from this analysis that affordable housing policies such as those in NYC are desirable. First, our analysis does not allow us to measure the costs that are imposed on landlords. Clearly, these policies primarily redistribute wealth and income from landlords to renters. The magnitude of the welfare losses imposed on landlords is largely unknown. Second, rent stabilization policies weaken the incentive to invest in housing. As a consequence, these policies have a significant negative impact on the long-term supply of both regulated and unregulated housing.

The main focus of this paper is on positive analysis. We provide a clean measure of the benefits that a renter obtains from living in an affordable or public housing unit. Our analysis provides a compelling explanation of why affordable housing policies are popular with the vast majority of urban renters and populist politicians. From the perspective of landowners and housing developers, affordable housing policies are undoubtedly costly. The main political advantage of affordable housing policies is that they allow local politicians to finance redistribution by implicitly taxing landlords and housing developers that benefit from rezoning policies. Since housing developers and landowners tend to benefit the most from improvements in urban quality via capitalization effects, affordable housing policies effectively redistribute part of these gains of urban redevelopment and improvement to low- and moderate-income renters. These renters tend to be the majority of voters in city elections.

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A Value Functions

The value function of a household with characteristics (w, y) that rents in the regulated market is given by:

$$\begin{aligned}
 V_r(y, w) &= p(w) 1\{y \leq \bar{y}\} \max\{v_p(y, r), v_r(y, r, 0), v_m(y, 0)\} \\
 &+ (1 - p(w)) 1\{y \leq \bar{y}\} \max\{v_r(y, r, w + 1), v_m(y, w + 1)\} \\
 &+ 1\{y > \bar{y}\} \max\{v_r(y, r, 0), v_m(y, 0)\}
 \end{aligned} \tag{17}$$

The value function of a household with characteristics (w, \bar{h}, y) that rents in the unregulated market is then given by:

$$\begin{aligned}
 V_m(y, \bar{h}, w) &= q_r p(w) 1\{y \leq \bar{y}\} \max\{v_p(y, m), v_r(y, m, 0), v_m(y, 0), v_s(y, \bar{h}, 0)\} \\
 &+ q_r(1 - p(w)) 1\{y \leq \bar{y}\} \max\{v_r(y, m, w + 1), v_m(y, w + 1), v_s(y, \bar{h}, w + 1)\} \\
 &+ q_r 1\{y > \bar{y}\} \max\{v_r(y, m, 0), v_m(y, 0), v_s(y, \bar{h}, 0)\} \\
 &+ (1 - q_r) p(w) 1\{y \leq \bar{y}\} \max\{v_p(y, m), v_m(y, 0), v_s(y, \bar{h}, 0)\} \\
 &+ (1 - q_r) (1 - p(w)) 1\{y \leq \bar{y}\} \max\{v_m(y, w + 1), v_s(y, \bar{h}, w + 1)\} \\
 &+ (1 - q_r) 1\{y > \bar{y}\} \max\{v_m(y, 0), v_s(y, \bar{h}, 0)\}
 \end{aligned} \tag{18}$$

Finally, the value function of a household living in public housing satisfies:

$$\begin{aligned}
 V_p(y) &= (1 - q_r) \max\{v_p(y, p), v_m(y, 0)\} \\
 &+ q_r \max\{v_p(y, p), v_r(y, p, 0), v_m(y, 0)\}
 \end{aligned} \tag{19}$$

B Law of Motions for the Income Distributions

The equilibrium rationing rule then implies the following law of motion for the stationary income distributions:

$$\begin{aligned}
g_p(y) &= k_p (1 - q_r) \int 1\{v_p(x, p) \geq v_m(x, 0)\} f(y|x) g_p(x) dx \\
&+ k_p q_r \int 1\{v_p(x, p) \geq \max[v_r(x, p, 0), v_m(x, 0)]\} f(y|x) g_p(x) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) p(w_j) \int_{x \leq \bar{y}} 1\{v_p(x, r) \geq \max[v_r(x, r, 0), v_m(x, 0)]\} f(y|x) g_r(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) (1 - q_r) \int \int_{x \leq \bar{y}} 1\{v_p(x, m) \geq \max[v_m(x, 0), v_s(x, \bar{h}, 0)]\} f(y|x) g_m(x, \bar{h}|w_j) dx d\bar{h} \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) q_r \int \int_{x \leq \bar{y}} 1\{v_p(x, m) \geq \max[v_r(x, m, 0), v_m(x, 0), v_s(x, \bar{h}, 0)]\} \\
&\times f(y|x) g_m(x, \bar{h}|w_j) dx d\bar{h}
\end{aligned} \tag{20}$$

Optimal amount of housing service for the mover in the unregulated market is given as a deterministic function of income and p_m .

$$h_m(p_m, y) = \underline{h} + \frac{\alpha(y - p_m \underline{h})}{p_m}$$

Since p_m is constant, we will drop it to simplify the notation.

$$\begin{aligned}
g_m(y, \bar{h}|0) &= k_p (1 - q_r) \int 1\{v_m(x, 0) \geq v_p(x, p)\} f(y|x) 1\{h_m(x) = \bar{h}\} g_p(x) dx \\
&+ k_p q_r \int 1\{v_m(x, 0) \geq \max[v_p(x, p), v_r(x, p, 0)]\} f(y|x) 1\{h_m(x) = \bar{h}\} g_p(x) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) p(w_j) \int_{x \leq \bar{y}} 1\{v_m(x, 0) \geq \max[v_p(x, r), v_r(x, r, 0)]\} f(y|x) 1\{h_m(x) = \bar{h}\} g_r(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) (1 - q_r) \int \int_{x \leq \bar{y}} 1\{v_m(x, 0) \geq \max[v_p(x, m), v_s(x, \bar{h}, 0)]\} \\
&\times f(y|x) 1\{h_m(x) = \bar{h}\} g_m(x, \bar{h}|w_j) dx d\bar{h} \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) (1 - q_r) \int_{x \leq \bar{y}} 1\{v_s(x, \bar{h}, 0) \geq \max[v_p(x, m), v_m(x, 0)]\} f(y|x) g_m(x, \bar{h}|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) q_r \int \int_{x \leq \bar{y}} 1\{v_m(x, 0) \geq \max[v_p(x, m), v_r(x, m, 0), v_s(x, \bar{h}, 0)]\} \\
&\times f(y|x) 1\{h_m(x) = \bar{h}\} g_m(x, \bar{h}|w_j) dx d\bar{h} \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) q_r \int_{x \leq \bar{y}} 1\{v_s(x, \bar{h}, 0) \geq \max[v_p(x, m), v_r(x, m, 0), v_m(x, 0)]\} \\
&\times f(y|x) g_m(x, \bar{h}|w_j) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x > \bar{y}} 1\{v_m(x, 0) \geq v_r(x, r, 0)\} f(y|x) 1\{h_m(x) = \bar{h}\} g_r(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) (1 - q_r) \int \int_{x > \bar{y}} 1\{v_m(x, 0) \geq v_s(x, \bar{h}, 0)\} f(y|x) 1\{h_m(x) = \bar{h}\} g_m(x, \bar{h}|w_j) dx d\bar{h} \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) (1 - q_r) \int_{x > \bar{y}} 1\{v_s(x, \bar{h}, 0) \geq v_m(x, 0)\} f(y|x) \\
&\times g_m(x, \bar{h}|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) q_r \int \int_{x > \bar{y}} 1\{v_m(x, 0) \geq \max[v_r(x, m, 0), v_s(x, \bar{h}, 0)]\} \\
&\times f(y|x) 1\{h_m(x) = \bar{h}\} g_m(x, \bar{h}|w_j) dx d\bar{h} \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) q_r \int_{x > \bar{y}} 1\{v_s(x, \bar{h}, 0) \geq \max[v_r(x, m, 0), v_m(x, 0)]\} f(y|x) g_m(x, \bar{h}|w_j) dx
\end{aligned} \tag{21}$$

and

$$\begin{aligned}
k_m g_m(0) &= k_p (1 - q_r) \int 1\{v_m(x, 0) \geq v_p(x, p)\} g_p(x) dx \\
&+ k_p q_r \int 1\{v_m(x, 0) \geq \max[v_p(x, p), v_r(x, p, 0)]\} g_p(x) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) p(w_j) \int_{x \leq \bar{y}} 1\{v_m(x, 0) \geq \max[v_p(x, r), v_r(x, r, 0)]\} g_r(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) (1 - q_r) \int \int_{x \leq \bar{y}} 1\{\max[v_m(x, 0), v_s(x, \bar{h}, 0)] \geq v_p(x, m)\} g_m(x, \bar{h}|w_j) dx d\bar{h} \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) q_r \int \int_{x \leq \bar{y}} 1\{\max[v_m(x, 0), v_s(x, \bar{h}, 0)] \geq \max[v_p(x, m), v_r(x, m, 0)]\} \\
&\times g_m(x, \bar{h}|w_j) dx d\bar{h} \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x > \bar{y}} 1\{v_m(x, 0) \geq v_r(x, r, 0)\} g_r(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) (1 - q_r) \int \int_{x > \bar{y}} g_m(x, \bar{h}|w_j) dx d\bar{h} \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) q_r \int \int_{x > \bar{y}} 1\{\max[v_m(x, 0), v_s(x, \bar{h}, 0)] \geq v_r(x, 0)\} g_m(x, \bar{h}|w_j) dx d\bar{h}
\end{aligned} \tag{22}$$

Moreover,

$$\begin{aligned}
g_r(y|0) &= k_p q_r \int 1\{v_r(x, 0) \geq \max[v_p(x, p), v_m(x, 0)]\} f(y|x) g_p(x) dx \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) p(w_j) \int_{x \leq \bar{y}} 1\{v_r(x, r, 0) \geq \max[v_p(x, r), v_m(x, 0)]\} f(y|x) g_r(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) q_r \int \int_{x \leq \bar{y}} 1\{v_r(x, m, 0) \geq \max[v_p(x, m), v_m(x, 0), v_s(x, \bar{h}, 0)]\} \\
&\times f(y|x) g_m(x, \bar{h}|w_j) dx d\bar{h} \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x > \bar{y}} 1\{v_r(x, r) \geq v_m(x, 0)\} f(y|x) g_r(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) q_r \int \int_{x > \bar{y}} 1\{v_r(x, m, 0) \geq \max[v_m(x, 0), v_s(x, \bar{h}, 0)]\} f(y|x) g_m(x, \bar{h}|w_j) dx d\bar{h}
\end{aligned} \tag{23}$$

and

$$\begin{aligned}
k_r g_r(0) &= k_p q_r \int 1\{v_r(x, 0, p) \geq \max[v_p(x, p), v_m(x, 0)]\} g_p(x) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) q_r \int \int_{x \leq \bar{y}} 1\{v_r(x, 0, m) \geq \max[v_p(x, m), v_m(x, 0), v_s(x, \bar{h}, 0)]\} \\
&\times g_m(x, \bar{h}|w_j) dx d\bar{h} \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) p(w_j) \int_{x \leq \bar{y}} 1\{v_r(x, r, 0) \geq \max[v_p(x, r), v_m(x, 0)]\} g_r(x|w_j) dx \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) q_r \int \int_{x > \bar{y}} 1\{v_r(x, m, 0) \geq \max[v_m(x, 0), v_s(x, \bar{h}, 0)]\} g_m(x, \bar{h}|w_j) dx d\bar{h} \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x > \bar{y}} 1\{v_r(x, r, 0) \geq v_m(x, 0)\} g_r(x|w_j) dx
\end{aligned} \tag{24}$$

$$\begin{aligned}
g_m(y, \bar{h}|w_j) &= k_r g_r(w_j - 1) \int_{x \leq \bar{y}} 1\{v_m(x, w_j) \geq v_r(x, r, w_j)\} f(y|x) 1\{h_m(x) = \bar{h}\} g_r(x|w_j - 1) dx \\
&+ k_m g_m(w_j - 1) (1 - q_r) \int_{x \leq \bar{y}} 1\{v_m(x, w_j) \geq v_s(x, \bar{h}, w_j)\} \\
&\times f(y|x) 1\{h_m(x) = \bar{h}\} g_m(x, \bar{h}|w_j - 1) dx d\bar{h} \\
&+ k_m g_m(w_j - 1) (1 - q_r) \int_{x \leq \bar{y}} 1\{v_s(x, \bar{h}, w_j) \geq v_m(x, w_j)\} f(y|x) g_m(x, \bar{h}|w_j - 1) dx \\
&+ k_m g_m(w_j - 1) q_r \int_{x \leq \bar{y}} 1\{v_m(x, w_j) \geq \max[v_s(x, \bar{h}, w_j), v_r(x, m, w_j)]\} \\
&\times f(y|x) 1\{h_m(x) = \bar{h}\} g_m(x, \bar{h}|w_j - 1) dx d\bar{h} \\
&+ k_m g_m(w_j - 1) q_r \int_{x \leq \bar{y}} 1\{v_s(x, \bar{h}, w_j) \geq \max[v_m(x, w_j), v_r(x, m, w_j)]\} f(y|x) g_m(x, \bar{h}|w_j - 1) dx
\end{aligned} \tag{25}$$

and

$$\begin{aligned}
g_m(w_j) &= g_m(w_j - 1) (1 - p(w_j - 1)) (1 - q_r) \int \int_{x \leq \bar{y}} g_m(x, \bar{h}|w_j - 1) dx d\bar{h} \\
&+ g_m(w_j - 1) (1 - p(w_j - 1)) q_r \int \int_{x \leq \bar{y}} 1\{\max[v_m(x, w_j), v_s(x, \bar{h}, w_j)] \geq v_r(x, m, w_j)\} \\
&\times g_m(x, \bar{h}|w_j - 1) dx d\bar{h} \\
&+ \frac{k_r}{k_m} g_r(w_j - 1) (1 - p(w_j - 1)) \int_{x \leq \bar{y}} 1\{v_m(x, w_j) \geq v_r(x, r, w_j)\} g_r(x|w_j - 1) dx
\end{aligned} \tag{26}$$

$$\begin{aligned}
g_r(y|w_j) &= k_r g_r(w_j - 1) \int_{x \leq \bar{y}} 1\{v_r(x, r, w_j) \geq v_m(x, w_j)\} f(y|x) g_r(x|w_j - 1) dx \\
&+ k_m g_m(w_j - 1) q_r \int \int_{x \leq \bar{y}} 1\{v_r(x, w_j, m) \geq \max[v_m(x, w_j), v_s(x, \bar{h}, w_j)]\} f(y|x) g_m(x, \bar{h}|w_j - 1) dx d\bar{h}
\end{aligned} \tag{27}$$

and

$$\begin{aligned}
g_r(w_j) &= g_r(w_j - 1) (1 - p(w_j - 1)) \int_{x \leq \bar{y}} 1\{v_r(x, r, w_j) \geq v_m(x, w_j)\} g_r(x|w_j - 1) dx \\
&+ \frac{k_m}{k_r} g_m(w_j - 1) (1 - p(w_j - 1)) q_r \int \int_{x \leq \bar{y}} 1\{v_r(x, m, w_j) \geq \max[v_m(x, w_j), v_s(x, \bar{h}, w_j)]\} \\
&\times g_m(x, \bar{h}|w_j - 1) dx d\bar{h}
\end{aligned} \tag{28}$$

C Flows for Rent-regulated Housing

The inflow from the rent-regulated market is given by:

$$IF_{rp}(w_j) = \int_{y \leq \bar{y}} 1\{v_p(y, r) \geq \max[v_r(y, r, 0), v_m(y, 0)]\} g_r(y|w_j) dy \tag{29}$$

Similarly, the voluntary flow of households out of rent-regulated housing is given by:

$$\begin{aligned}
OF_r &= k_r \sum_{j=0}^{\infty} p(w_j) g_r(w_j) \int_{y \leq \bar{y}} 1\{v_r(y, r, 0) \leq \max[v_p(y, r), v_m(y, 0)]\} g_r(y|w_j) dy \\
&+ k_r \sum_{j=0}^{\infty} (1 - p(w_j)) g_r(w_j) \int_{y \leq \bar{y}} 1\{v_m(y, w_j + 1) \geq v_r(y, r, w_j + 1)\} g_r(y|w_j) dy \\
&+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{y > \bar{y}} 1\{v_m(y, 0) \geq v_r(y, r, 0)\} g_r(y|w_j) dy
\end{aligned} \tag{30}$$

Note that the first term is the outflow of those households that have an offer to move into public housing. The second term is the outflow of households eligible for public housing who do not have an offer to move into public housing. The last term is the outflow of households above the eligibility threshold to unregulated housing. The flow into rent-regulated housing is given by:

$$\begin{aligned}
IF_r &= k_p q_r \int 1\{v_r(y, p, 0) \geq \max[v_p(y, p), v_m(y, 0)]\} g_p(y) dy \\
&+ k_m \sum_{j=0}^{\infty} p(w_j) g_m(w_j) q_r \int \int_{y \leq \bar{y}} 1\{v_r(y, m, 0) \geq \max[v_p(y, m), v_m(y, 0), v_s(y, \bar{h}, 0)]\} \\
&\times g_m(y, \bar{h}|w_j) dy d\bar{h} \\
&+ k_m \sum_{j=0}^{\infty} (1 - p(w_j)) g_m(w_j) q_r \int \int_{y \leq \bar{y}} 1\{v_r(y, m, w_j + 1) \geq \max[v_m(y, w_j + 1), v_s(y, \bar{h}, w_j + 1)]\} \\
&\times g_m(y, \bar{h}|w_j) dy d\bar{h} \\
&+ k_m \sum_{j=0}^{\infty} g_m(w_j) q_r \int \int_{y > \bar{y}} 1\{v_r(y, 0, m) \geq \max[v_m(y, 0), v_s(y, \bar{h}, 0)]\} g_m(y, \bar{h}|w_j) dy d\bar{h}
\end{aligned} \tag{31}$$

D Model Extensions

Let (k_{ip}, k_{ir}, k_{im}) denote the relevant type specific market shares. Let $g_{im}(w)$ ($g_{ir}(w)$) denote the marginal distribution of wait times for households of type i in unregulated (rent-regulated) housing in stationary equilibrium. Let $g_{ip}(y)$ denote the density of income of households of type i that are inside public housing at the beginning of each period. Similarly let $g_{im}(y|w)$ ($g_{ir}(y|w)$) denote the stationary density of income conditional on wait time for households in the unregulated (regulated) market.

The voluntary flow of type i households out of public housing is given by:

$$\begin{aligned}
 OF_{ip} &= k_{ip} (1 - q_r) \int 1\{v_{im}(y, 0) > v_{ip}(y, p)\} g_{ip}(y) dy \\
 &+ k_{ip} q_r \int 1\{v_{im}(y, 0) \geq \max[v_{ip}(y, p), v_{ir}(y, p, 0)]\} g_{ip}(y) dy \\
 &+ k_{ip} q_r \int 1\{v_{ir}(y, p, 0) \geq \max[v_{ip}(y, p), v_{im}(y, 0)]\} g_{ip}(y) dy
 \end{aligned} \tag{32}$$

Note that the first two terms is the outflow to the unregulated market and the third term captures the outflow to the rent-regulated market. The flow into public housing of type i households is given by:

$$\begin{aligned}
 IF_{ip} &= p(\bar{w}) [k_{im} g_{im}(\bar{w}) IF_{imp}(\bar{w}) + k_{ir} g_{ir}(\bar{w}) IF_{irp}(\bar{w})] \\
 &+ [k_{im} g_{im}(\bar{w} + 1) IF_{imp}^i(\bar{w} + 1) + k_{ir} g_{ir}(\bar{w} + 1) IF_{irp}(\bar{w} + 1)]
 \end{aligned} \tag{33}$$

where the inflow from the unregulated market conditional on wait time is:

$$\begin{aligned}
 IF_{imp}(w_j) &= (1 - q_r) \int \int_{y \leq \bar{y}} 1\{v_{ip}(y, m) \geq \max[v_{im}(y, 0), v_{is}(y, \bar{h}, 0)]\} \\
 &\times g_{im}(y, \bar{h}|w_j) dy d\bar{h} \\
 &+ q_r \int \int_{y \leq \bar{y}} 1\{v_{ip}(y, m) \geq \max[v_{ir}(y, 0), v_{im}(y, 0), v_{is}(y, \bar{h}, 0)]\} g_{im}(y, \bar{h}|w_j) dy d\bar{h}
 \end{aligned} \tag{34}$$

and the inflow from the rent-regulated market is given by:

$$IF_{irp}(w_j) = \int_{y \leq \bar{y}} 1\{v_{ip}(y, r) \geq \max[v_{ir}(y, r, 0), v_{im}(y, 0)]\} g_{ir}(y|w_j) dy \tag{35}$$

Equilibrium in public housing requires that for each housing type i , we have

$$IF_p = \sum_{i=1}^I IF_{ip} = \sum_{i=1}^I OF_{ip} = OF_p \tag{36}$$

Next consider the market for regulated housing. The voluntary flow of type i households out of rent-

regulated housing is given by:

$$\begin{aligned}
OF_{ir} &= k_{ir} \sum_{j=0}^{\infty} p(w_j) g_{ir}(w_j) \int_{y \leq \bar{y}} 1\{v_{ip}(y, r) \leq \max[v_{ir}(y, r, 0), v_{im}(y, 0)]\} g_{ir}(y|w_j) dy \\
&+ k_{ir} \sum_{j=0}^{\infty} p(w_j) g_{ir}(w_j) \int_{y \leq \bar{y}} 1\{v_{im}(y, 0) \geq \max[v_{ip}(y, r), v_{ir}(y, r, 0)]\} g_{ir}(y|w_j) dy \\
&+ k_{ir} \sum_{j=0}^{\infty} (1 - p(w_j)) g_{ir}(w_j) \int_{y \leq \bar{y}} 1\{v_{im}(y, w_j + 1) \geq v_{ir}(y, r, w_j + 1)\} g_{ir}(y|w_j) dy \\
&+ k_{ir} \sum_{j=0}^{\infty} g_{ir}(w_j) \int_{y > \bar{y}} 1\{v_{im}(y, 0) \geq v_{ir}(y, r, 0)\} g_{ir}(y|w_j) dy
\end{aligned} \tag{37}$$

Note that the first term is the outflow to public housing. The second term is the outflow to unregulated housing if you have an offer to move into public housing. The last two terms are the outflow to unregulated housing if you do not have an offer to move into public housing.

The flow into rent-regulated housing is given by:

$$IF_{ir} = k_{im} \sum_{j=0}^{\infty} g_{im}(w_j) IF_{imr}(w_j) + k_{ip} IF_{ipr} \tag{38}$$

where the inflow from the unregulated market conditional on wait time is:

$$\begin{aligned}
IF_{imr}(w_j) &= q_r p(w_j) \int \int_{y \leq \bar{y}} 1\{v_{ir}(y, m, 0) \geq \max[v_{ip}(y, m), v_{im}(y, 0), v_{is}(y, \bar{h}, 0)]\} \\
&\times g_{im}(y, \bar{h}|w_j) dy d\bar{h} \\
&+ q_r (1 - p(w_j)) \int \int_{y \leq \bar{y}} 1\{v_{ir}(y, m, w_j + 1) \geq \max[v_{im}(y, w_j + 1), v_{is}(y, \bar{h}, w_j + 1)]\} \\
&\times g_{im}(y, \bar{h}|w_j) dy d\bar{h} \\
&+ q_r \int \int_{y > \bar{y}} 1\{v_{ir}(y, 0, m) \geq \max[v_{im}(y, 0), v_{is}(y, \bar{h}, 0)]\} g_{im}(y, \bar{h}|w_j) dy d\bar{h}
\end{aligned} \tag{39}$$

and the flow from public housing market to rent-regulated housing is given by:

$$IF_{ipr} = q_r \int 1\{v_{ir}(y, p, 0) \geq \max[v_{ip}(y, p), v_{im}(y, 0)]\} g_{ip}(y) dy \tag{40}$$

Equilibrium requires that the aggregate outflow equal the aggregate inflow

$$IF_r = \sum_{i=1}^I IF_{ir} = \sum_{i=1}^I OF_{ir} = OF_r \tag{41}$$

As before, we can define a stationary equilibria with rationing as follows:

Definition 2 A stationary equilibrium with rationing for the extended model consists of the following: a) market shares (k_{ip}, k_{ir}, k_{im}) $i = 1, \dots, I$, b) offer probability $p(w)$ and q_r , c) distributions $g_{ip}(y)$, $g_{im}(w)$, $g_{ir}(w)$, $g_{im}(y|w)$, and $g_{ir}(y|w)$, and d) value functions $V_{ip}(y)$, $V_{im}(y, w)$ and $V_{ir}(y, w)$, such that:

1. Households behave optimally and value functions satisfy the equations above.
2. The housing authority behaves according the administrative rules described above.
3. The densities are is consistent with the laws of motion and optimal household behavior.
4. $p(w)$ satisfies the market clearing condition for public housing:

$$OF_p = IF_p \tag{42}$$

5. q_r satisfies the market clearing condition for rent-regulated housing:

$$OF_r = IF_r \tag{43}$$

6. The following identities hold for the market shares:

$$\begin{aligned} \sum_{i=1}^I k_{ir} &= k_r \\ \sum_{i=1}^I k_{im} &= k_m \\ k_{ip} + k_{ir} + k_{im} &= s_i \quad i = 1, \dots, I \end{aligned} \tag{44}$$

It is fairly straightforward to extend the law of motions for the equilibrium densities.³¹

E Distribution of Housing Quality

Using the estimates in Column II of Table 2 we can predict the quality of each housing unit in the regulated market. Figure 8 plots the histogram of the quality distribution. In the quantitative model, we approximate this distribution by two discrete types. This is mainly done for computational reasons. It is, in principle, possible to estimate models with more than two types.

³¹An appendix is available upon request from the authors that provides the relevant equations.

Figure 8: Distribution of Housing Quality in the Regulated Market

